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論文題目　：Evaluating the implementation of innovative technology in Japan's bidding system: a dynamic Stackelberg game theoretical analysis
（邦訳：日本の入札制度における革新的技術導入の評価：動学的 Stackelberg ゲーム理論による分析）
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要旨（邦訳）：
建設分野におけるイノベーションは、労働生産性の向上にとって重要である。適切な調達戦略は、技術革新の促進に大きく寄与する。日本の総合評価落札方式（Comprehensive Evaluation Method: CEM）は、多数の評価要素を含む入札制度であり、ダンピング入札を防止するために「調査基準価格」と呼ばれる実質的な下限価格（Lower Bound: LB）が設定されている。しかしながら、この下限価格は、コスト削減を伴う革新的技術の導入を抑制する可能性がある。現在のところ、この下限価格が入札制度に与える影響を評価する方法や、イノベーションを促進するための最適なパラメータを特定する手法は十分に確立されていない。
本研究の目的は、公共発注者および革新的技術を有する企業の双方にとっての利益を最大化する最適なパラメータ値を導出するための分析枠組みを構築するとともに、革新的技術の導入に関する現状を踏まえた最適なシナリオを提示することである。本研究では、公共発注者と入札者の逐次的意思決定をモデル化するために、Stackelberg ゲーム理論に基づく枠組みを構築し、これを最適化問題として定式化した。
分析の結果、（1）本手法は局所最適性条件を満たす最適解を効果的に導出できること、（2）実質的な下限価格を設けない総合評価落札方式では、落札価格が低下し、革新的企業の落札確率および利益が増加する可能性があることが明らかとなった。本研究で提案するモデルは、建設分野におけるイノベーション促進に寄与するとともに、入札制度設計に関する新たな知見を提供するものである。従来の多パラメータ型入札制度に関する研究の多くが単一の目的関数に焦点を当てているのに対し、本研究では異なる目的を持つ二者（公共発注者と企業）を同時に分析対象とすることで、多パラメータ型入札制度の設計に対してより実務的な視点を提示している。
＜キーワード＞
総合評価落札方式、ゲーム理論モデル、建設イノベーション、公共調達、日本

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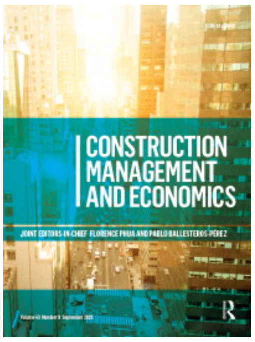

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Evaluating the implementation of innovative technology in Japan's bidding system: a dynamic Stackelberg game theoretical analysis

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ABSTRACT

Innovation in construction is crucial for enhancing labour productivity. Effective procurement strategies promote technological advancements. Japan's Comprehensive Evaluation Method (CEM) includes a multi-parameter bidding system incorporating a substantial lower bound (LB) known as 'Chosa Kijun Kakaku' to prevent underbidding. However, this bound can limit innovative practices that reduce costs. Currently, there are no methods to evaluate the impact of this bound or identify optimal parameters to encourage innovation. This study aims to develop a framework for determining the optimal parameter values to maximize benefits for public owners and innovative firms while also deriving an optimal scenario for the current circumstances regarding innovative technology adoption. A Stackelberg game theoretical framework models the sequential decisions of public owners and bidders, framing it as an optimisation problem. Findings include: (i) the method effectively identifies optimal solutions that meet local optimality conditions; (ii) a CEM without substantial LB results in lower award prices, higher winning probabilities, and increased profits for an innovative company. This model fosters innovation and provides new insights into bidding design. Unlike most multi-parametric bidding research that centres on a single objective function, this study investigates two parties with differing objectives, offering a practical perspective on multi-parameter bidding design.

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Multi-parameter bidding method; game-theoretic model; construction innovation; public procurement; Japan

Introduction

Innovation drives industrial development. In recent years, the construction sector has witnessed technological advancements, including Building Information Modelling (BIM) for design and robotics for site inspections. With a declining working-age population in Japan's construction industry (MLIT 2021a), enhancing innovation capabilities in industrial technology to boost productivity has become a crucial focus since 2016 (MLIT 2016). To reduce both the number of workers and their working hours, contractors are encouraged to implement information communication technology (ICT), such as BIM, Artificial Intelligence (AI), the Internet of Things (IoT), Digital Twins (DTs), robotics and labour-saving technologies like fast-erecting cranes in the construction process. However, challenges such as high upfront costs (Ahmed 2018, Demirkesen and Tezel 2022), shortages of skilled workers, and training gaps (Demirkesen and Tezel 2022, Olatunde *et al.* 2022, Yap *et al.* 2022, Oke *et al.* 2024), cybersecurity and data management issues including data breaches and sabotage (Wang *et al.* 2024), and a lack of reliable communication systems (Stas and Abrishami 2024) hinder collaboration among stakeholders due to the industry's fragmented nature (Lu and Korman 2010, Korman and Lu 2011, Lee and Kim 2017, Demirkesen and Tezel 2022, Gao *et al.* 2024), obstructing construction companies from innovating through technology. It is noteworthy that the construction sector has historically been conservative, with many companies hesitant to adopt new technologies or modify established workflows (Qiu *et al.* 2019, Wang and Chen 2023).

Furthermore, regulatory and legal complexities hinder innovation within the industry. In Japan, the construction industry operates under a top-down model led by the Ministry of Land, Infrastructure, Transport and Tourism (MLIT), which makes regulations and laws essential for fostering innovation by adopting new

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technologies. Encouraging new technologies, including ICT applications, as an evaluation criterion in the procurement method is essential.

Two main procurement methods are *pure price competitive bidding* and a *comprehensive evaluation method* (CEM). Both methods impose higher (the ceiling price[1]) and substantial lower bound (LBs).[2] CEM, a multi-parameter bidding process introduced in 2005, is typically used for projects procured by the national government (Kinoshita *et al.* 2008). CEM takes into account bid prices and technical factors such as experience, past performance and proposals. Points are assigned to these technical factors, typically with the winning bidder achieving the highest ratio of technical points to the bid price (Suzuki and Horita 2014). CEM aims to foster a healthy public works market, ensure project quality and prevent bid dumping. Public owners also intend to utilize CEM to encourage construction firms to adopt new technologies; if a contractor plans to employ ICT as requested or has relevant experience, they will be awarded technical evaluation points (MLIT 2018). The current CEM includes an investigative criterion price, the '*Chosa Kijun Kakaku*,' the substantial LB determined by a reference model, serving as an additional measure of bid realism. Additional documentation is required to prevent bid dumping if a bid falls below this price.

Although substantial LBs prevent dumping bidding, they may hinder innovative practices that could reduce costs in the Japanese construction industry. Since information disclosure is necessary to maintain a fair procurement process, most information required to determine the ceiling price and the LB is accessible to all bidders. Therefore, in many cases, bidders can accurately estimate both the ceiling price and the LB. Some public owners deduct technical evaluation points from bids that fall below the substantial LB, even when bidders successfully demonstrate the legitimacy of their proposals. This is partly due to such bids requiring additional administrative work. The Japanese construction industry prioritizes quality and generally has a cultural aversion to low bids, often seen as linked to potential compromises in quality. Under these conditions, construction firms that innovate by introducing new technologies capable of reducing costs often find their bids rejected. Therefore, the awarded price does not decrease with substantial LBs in place because many bidders propose bids at the LB or above it (Figure 1). Consequently, the outcome of all this is that the current procurement method for public works is ineffective in adapting to changes related to innovation, thus discouraging innovative

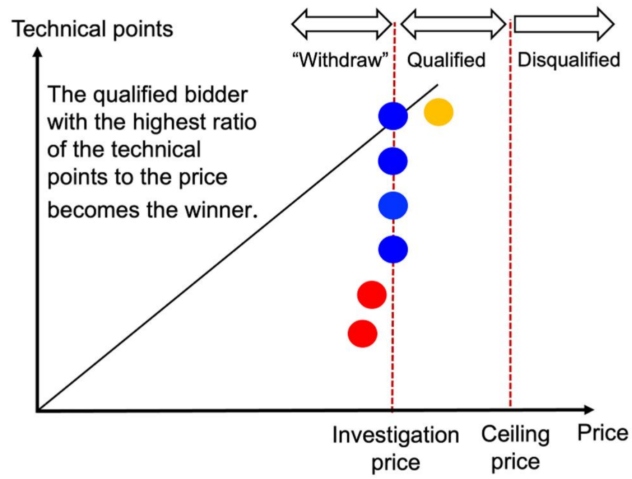

Figure 1. Scenario with LB: CEM – Technical points determine the winner.

bidders from submitting tenders. The shortcomings of LBs in Japan have been studied theoretically (Iwamatsu et al. 2013, Suzuki and Horita 2014, Yamaki and Yabuki 2018, Nishida and Horida 2019); however, a specific improvement method for multi-parameter bidding has not yet been developed.

To address the previously mentioned issue, a strategic approach is needed: (a) develop a framework for determining the optimal parameter values in multi-parameter bidding—how to add technical points to introduce new technologies and how to evaluate its cost reduction; (b) derive an optimal scenario for technology adoption by comparing the scenarios with substantial LB and without LB; and (c) implement new dumping prevention measures, which is a later study. Regarding (a) and (b), however, no study has examined or compared the differences in awarded prices, winning probability and profit of an innovative company before and after removing the LB. Additionally, the necessary conditions to establish the optimal parameter values that maximise benefits for both public owners and innovative companies after removing the LB remain unexplored. The classical Stackelberg game, which models competition in a duopoly market with one leader and one follower (Stackelberg 1934), can effectively inform the design of this bidding method within the field of construction innovation. The optimal solution can be achieved through an equilibrium where the innovative company maximises its profits based on the public owner's initial actions. In contrast, the public owner minimises costs based on the innovative company's decisions.

Given this context, the present study focussed on (a) and (b) in the aforementioned strategic approach and aimed to achieve the objectives: (1) modelling the current CEM (with the LB) alongside the CEM without the LB to maximise profits for an innovative company and minimise bidding costs for a public owner using a Stackelberg game to develop a framework for determining the optimal parameter values in multi-parameter bidding; (2) comparing the differences in awarded prices for a public owner, as well as the winning probability and profit of the innovative company, before and after the removal of the LB. This article provides a method for designing multi-parameter bidding in construction innovation. It also offers practical insights into how the proposed procurement method contributes to introducing new technologies. Moreover, it suggests designing a bidding method for construction innovations in other countries.

Another paper explores the mechanism for preventing dumping bids after removing lower bounds. This research analyses the current scenario with a single innovative company operating in a specific region. As the diffusion of new technologies grows, it is crucial to consider a model involving multiple innovative companies. Additionally, developing a multi-parameter bidding strategy that promotes adopting new technologies must include the potential for future cost reductions. Future research will also focus on a strategic model for disseminating new technology and establishing its market, which entails timely adjustments to a set of optimal parameter values.

Literature review

Bidding is a global assessment of the trade-offs between clients and contractors. From the contractor's perspective, competitive bidding has been studied since the work of Friedman (1956) and Gates (1967). In economic terms, construction contract bidding is often viewed as a common value auction. Consequently, competitive bidding models rooted in auction theory have been created to determine optimal markups mitigate the winner's curse (Skitmore and Pemberton 1994, Dyer and Kagel 1996, Skitmore and Cattell 2013, Skitmore 2014a, Ahmed and El-adaway 2023), and predict winning probabilities (Skitmore 2002, 2004), as well as to enhance a contractor's competitive position for long-term engagements with other firms (Kim and Reinschmidt 2006). Utility theory has also been applied in construction bidding to analyse win probabilities, free from auction theory's assumptions (Chou et al. 2013). While Marzouk and Moselhi (2003) proposed a model that considers two goals to help contractors estimate markups and aid owners in assessing bid proposals through utility theory, the optimisation of a contractor's markup alongside a client's awarded price by modifying parameters continues to be an area needing further exploration.

From a client's perspective, multi-parameter bidding methods have significantly grown to assist clients in selecting a suitable bidder. The foundational A + B (cost + time) bidding method, a staple in infrastructure projects, has shown remarkable effectiveness across numerous case studies, consistently achieving reduced project durations with minimal cost increases (Herbsman 1995, Lambropoulos 2007, Gupta et al. 2015). Building on this success, the A + B + I/D (incentive/disincentive) model (Shr et al. 2000, 2004) and the innovative tri-parameter models that consider risk factors (El-Sayegh and Rabie 2016, Rabie and El-Sayegh 2017) highlight how multi-parameter approaches enhance decision-making by addressing critical issues like float loss and schedule delays. Moreover, as we move toward a more sustainable future, the emergence of models such as A + S (cost + sustainability) (El-Sayegh et al. 2022) and A + B + C (cost + time + environmental cost) (Ahn et al. 2013) emphasises the growing importance of environmental and social criteria in the bidding process. Nonetheless, in the field of construction innovation, there is currently a lack of research focussed on evaluating contractors based on the application of new technology parameters.

Consequently, earlier studies have emphasised the development of metrics focussed on a single objective, whether it assists public or private owners in contractor selection (e.g., El-Sayegh and Rabie 2016, Mohamed et al. 2022) or enables contractors to compete with one another with the best fitting model (Cheng et al. 2011, Skitmore 2014b). Furthermore, no parameter related to the application of new technology has been accounted for in the existing multi-parameter bidding method. To foster technological innovation and establish a mutually beneficial scenario between a public owner and an innovative contractor, it is essential to model their interactions by introducing a new technology evaluation metric that optimises both the innovative contractor's profits and the public owner's costs, thereby promoting the adoption of new technology.

Methodology

The imperative of the Stackelberg game for the analysis of procurement design

A Stackelberg game typically features two primary players: a leader and a follower. Each player seeks to optimise their distinct and often conflicting goals. The leader initiates by selecting a strategy, while the follower responds with the optimal choice based on the leader's action. The leader can anticipate the follower's reaction by presuming that the follower consistently seeks to optimise their strategy according to the leader's approach. By understanding the follower's potential options, the leader can enhance the objective function by adjusting their strategy. The solution is a subgame perfect Nash equilibrium, determined through backward induction (Stackelberg 1934). The Stackelberg game has proven effective in examining the interaction between public owners and contractors in the construction industry to optimise design changes (Liu *et al.* 2022), design payment mechanisms in PPP projects (Shang and Abdel Aziz 2020) and optimize profit and time (Hafezalkotob 2018).

Despite the valuable insights from these studies, no Stackelberg games have addressed the bidding evaluation problem. Nevertheless, such games are well-suited for analysing these issues, with the public owner, as the leader, always making the first move. Contractors, as followers, can then maximise their profits based on the owner's decisions, after which the owner minimises costs based on the followers' actions. Compared to previous multi-parameter bidding studies, this dynamic game provides a clearer understanding of the interactions between public owners and contractors in the procurement process.

Proposed Stackelberg game model for multi-parameter bidding

In the proposed Stackelberg model, a public owner makes the first move and reveals its point calculation method for a comprehensive evaluation to bidders: the technology evaluation coefficient, represented by β, and the weight associated with a bidding price point, represented by γ. Following the comprehensive evaluation rule, bidders decide on their price offers, including the profits they want to obtain (α) and the extent to which they want to use technology (t). Given that this study centres on the interaction between a public owner and bidders (both innovative and conservative companies), the game is played sequentially. It seeks the maximum profit for an innovative company and the lowest bidding cost for a public owner. To apply the Stackelberg game, the public owner needs the cost information of bidders. As explained later, however, this situation is not unrealistic in Japan. Figure 2 shows the decision-making variables.

where

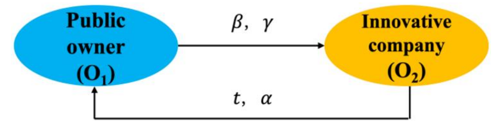

Figure 2. Decision-making variables in the game.

β: Evaluation coefficient of new technologies in the CEM ($\beta \geq 0$);

γ: Weight associated with a price point in the CEM ($\gamma \geq 0$);

t: Extent of technology introduction t of an innovative company ($t > 0$);

α: Profit level secured by the innovative company (JPY) ($\alpha > 0$).

The public owner supplies information regarding the technology evaluation rule, denoted by β, for the technology employed by bidding companies to support new technology applications. In the public construction market, two types of companies operate: 'innovative companies' that actively introduce innovative technologies, such as ICT, and the rest $(n-1)$ that use conventional technologies (referred to here as 'conventional companies').

Because conventional companies use traditional technologies, t is zero for all such enterprises. Two scenarios were analysed. Figure 1 (Scenario with LB) is the current multi-parameter bidding method. Here, all bidders are assumed to bid at the substantial LB in this scenario (only bids coloured in blue are analysed in Figure 1). Figure 3 (Scenario without LB) is the CEM without substantial LB method. This method intends to encourage the bidder, who demonstrates that technological innovation is the reason that enables the reduction of costs. There are two noteworthy points: first, all bidders are assumed to bid at the LB except the innovative company in Scenario without LB; second, the CEM without substantial LB requires infrastructure as an avenue through which to prevent dumping. We have been studying this infrastructure as a future direction.

Analysis flow

Figure 4 shows the flow of the analysis. Therefore, this study aimed to minimise the costs incurred by a public owner (function Min O_1) and maximise the profits an innovative company earns (function Max O_2). To this end, an innovative company that will win a bid (Figure 3) was explored. We sought a combination of β, γ, t and α that minimises an owner's project cost and maximises an innovative company's expected profit.

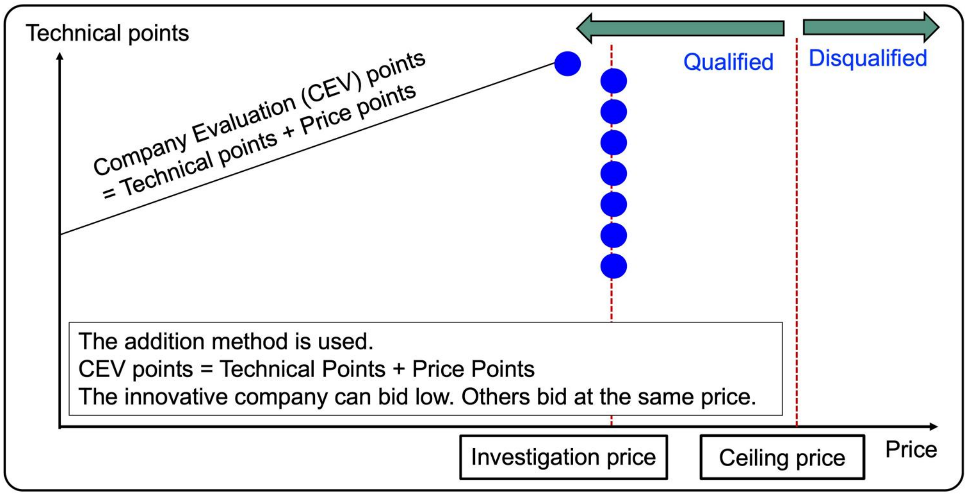

Figure 3. Scenario without LB: CEM with no substantial lower bound (Both price and technical points determine the winner).

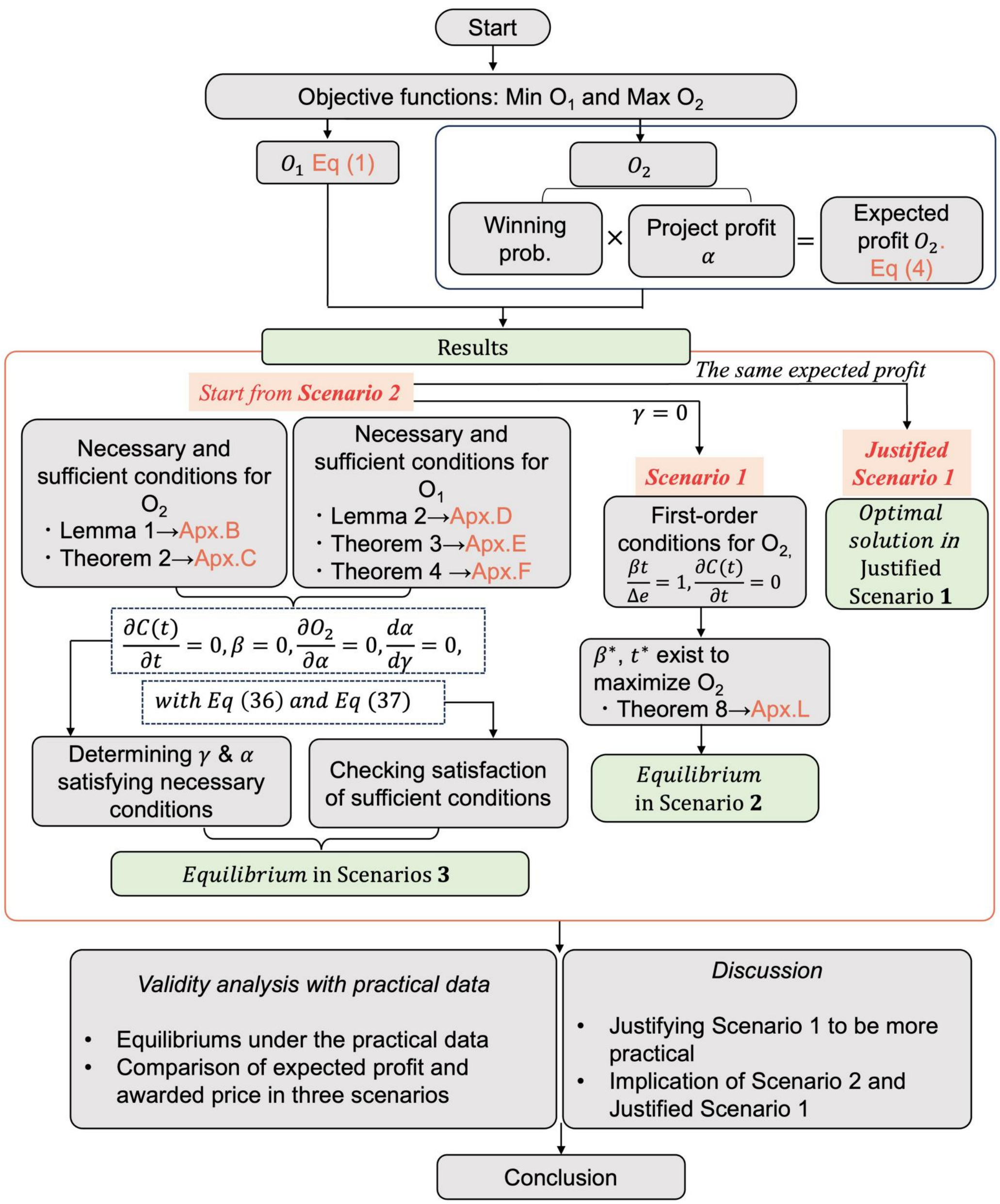

Figure 4. Flow of the analysis.

We first analyse the Scenario without LB. The equilibrium solution was obtained by analysing the necessary and sufficient conditions for O_2 and O_1; the equilibrium solution was obtained. In the Scenario with LB, the price evaluation weight, γ, is zero, and the same analysis method was applied to obtain the equilibrium solution. Since the Scenario with LB has a single objective function of the innovative company, O_2, a solution unrealistically advantageous to the innovative company was obtained. A Justified Scenario was introduced to derive a realistic solution and compare its performance with the Scenario with LB by adding the condition that the innovative company obtained the same expected profit from the Scenario without LB.

Model development

Overview of the model

Each scenario aims to minimise the owner's contract price and maximise the innovative company's expected profit.

The objective function for public owners (O_1)

Scenario without LB analyses the effect of innovative technologies. Thus, O_1 becomes the contract price with the innovative company in this scenario. The objective function of the owner is given by

$$\text{Min } O_1 = b(t(\beta,\gamma),\alpha(\beta,\gamma)) = C(t(\beta,\gamma)) + \alpha(\beta,\gamma) \tag{1}$$

where the bid price is set as

$$b(t,\alpha) = C(t(\beta,\gamma)) + \alpha(\beta,\gamma) \tag{2}$$

$$C(t(\beta,\gamma)) = I(t(\beta,\gamma)) + d(t(\beta,\gamma)) \tag{3}$$

where $C(t(\beta,\gamma)$ is construction cost, $\alpha(\beta,\gamma)$ denotes project profit, $I(t(\beta,\gamma))$ represents production cost and $d(t(\beta,\gamma))$ is depreciation cost of equipment or its rental costs.

The objective function of the contractor (O_2)

The objective function of the innovative company is given by

$$\text{Max } O_2 = \alpha(\beta,\gamma)*Prob(t(\beta,\gamma),\alpha(\beta,\gamma)) \tag{4}$$

where *Prob* is the probability of winning for the innovative company.

Each company's evaluation value, x_j, is represented by

$$x_j = \beta t_j + \gamma\frac{\overline{P}-b_j}{\overline{P}-\underline{P}} + \varepsilon_j \tag{5}$$

where x_1 is an innovative company's evaluation value, $x_j (j = 2, \ldots, n)$ is a conventional company's evaluation value, ε_j denotes the company/engineer evaluation points of company j $(j = 1, \ldots, n)$, in which $j = 1$ for an innovative company and $j = 2, \ldots, n$ for conventional companies, $\overline{P}$ is the ceiling price (JPY), $\underline{P}$ refers to the substantial LB (JPY), b_j represents the bid price of the jth company (JPY) and t pertains to the extent of technology introduction in the innovative company. This yields

$$t_j = \begin{cases} t > 0, & j = 1 \\ 0, & j = 2, \ldots, n \end{cases} \tag{6}$$

The addition method of the CEM was used instead of the division approach which Japanese public owners commonly use as it cannot define price weighting independently of a technical evaluation point.

(1) An innovative company's evaluation value x_1 is given by

$$\begin{aligned} x_1 &= \beta t(\beta,\gamma) + \gamma\frac{\overline{P}-b}{\overline{P}-\underline{P}} + \varepsilon_1 \\ &= \beta t(\beta,\gamma) + \gamma\frac{\overline{P}-\{C(t(\beta,\gamma))+\alpha(\beta,\gamma)\}}{\overline{P}(1-s)} + \varepsilon_1 \end{aligned} \tag{7}$$

$$s = {}^{\underline{P}}/_{\overline{P}} \tag{8}$$

(2) A conventional company does not introduce new technology ($t = 0$) and bids at the substantial LB ($b_j = \underline{P}$). According to Equation (5), its evaluation value $x_j (j = 2, \ldots, n)$ is given by

$$x_j = \gamma + \varepsilon_j \tag{9}$$

Expected profit and award price of an innovative company

- The innovative company's winning probability

This section presents the analysis of a more complicated Scenario without LB.

Theorem 1. The innovative company's probability of winning is as follows:

(1) Scenario without LB: Both technical points and price points are included.

$$Prob = \frac{1}{n}\left[1 - \left(\frac{CE(t,\alpha,\beta,\gamma)}{\Delta e}\right)^n\right] + \frac{CE(t,\alpha,\beta,\gamma)}{\Delta e} \tag{10}$$

provided

$$0 \le \frac{CE(t,\alpha,\beta,\gamma)}{\Delta e} \le 1 \tag{11}$$

(2) Scenario with LB: The CEM: All bidders bid at the substantial LB. Technical points determine the winner ($\gamma = 0$).

$$Prob = \frac{1}{n}\left[1 - \left(\frac{\beta t(\beta)}{\Delta e}\right)^n\right] + \frac{\beta t(\beta)}{\Delta e} \tag{12}$$

where

$$CE(t,\alpha,\beta,\gamma) = \beta t(\beta,\gamma) + \gamma \frac{\overline{P}s - \{C(t(\beta,\gamma)) + \alpha(\beta,\gamma)\}}{\overline{P}(1-s)} \tag{13}$$

Proof: See Appendix A for detail.

- The formula for the expected profit of an innovative company (O_2)

This section recounts the derivation of the innovative company's expected profit and award price for further analysis. As mentioned, this company intends to maximise its expected profit, whereas the owner seeks to minimise the award price. As in the previous section, the analysis starts from the more complicated scenario, the Scenario without LB.

First, $CE(t,\alpha,\beta,\gamma)$ is transformed as

$$CE(t,\alpha,\beta,\gamma) = \beta t(\beta,\gamma) + \gamma \frac{\overline{P}s - C(t(\beta,\gamma))}{\overline{P}(1-s)} - \gamma \frac{\alpha(\beta,\gamma)}{\overline{P}(1-s)} \tag{13'}$$

Setting

$$\delta(t(\beta,\gamma),\beta,\gamma) = \beta t(\beta,\gamma) + \gamma \frac{\overline{P}s - C(t(\beta,\gamma))}{\overline{P}(1-s)} \tag{14}$$

$$h = \frac{-1}{\overline{P}(1-s)} \tag{15}$$

Equations (13′) and (14) can be further rewritten as

$$CE(t,\alpha,\beta,\gamma) = \delta(t(\beta,\gamma),\beta,\gamma) + g(\alpha(\beta,\gamma),\gamma) \tag{16}$$

$$\delta(t(\beta,\gamma),\beta,\gamma) = \beta t(\beta,\gamma) + h\gamma(C(t(\beta,\gamma)) - \overline{P}s) \tag{17}$$

$$g(\alpha(\beta,\gamma),\gamma) = h\alpha(\beta,\gamma)\gamma \tag{18}$$

For the time being, we denote $t(\beta,\gamma)$, $\alpha(\beta,\gamma)$, $\delta(t(\beta,\gamma),\beta,\gamma)$ and $g(\alpha(\beta,\gamma),\gamma)$ by t, α, δ and g, respectively. Equations (13′), (17) and (18) are simply denoted as

$$CE(t,\alpha,\beta,\gamma) = \delta + g \tag{16'}$$

$$\delta = \beta t + h\gamma(C(t)) - \overline{P}s) \tag{17'}$$

$$g = \text{h}\alpha\gamma \tag{18'}$$

Then, the expected profit of the innovative company, O_2, is given by

$$O_2 = \alpha(\beta,\gamma) * Prob = \alpha * Prob$$
$$= \frac{\alpha}{n}\left\{1 - \left(\frac{\delta+g}{\Delta e}\right)^n\right\} + \alpha\left(\frac{\delta+g}{\Delta e}\right)$$
$$= \frac{\alpha}{n}\{1 - x^n\} + \alpha x \tag{19}$$

where

$$x = \frac{\delta+g}{\Delta e} \tag{20}$$

Equation (11) is rewritten as

$$0 \le x \le 1 \tag{11'}$$

Characteristics of this model and assumptions

The formulation of this model does not fall within the realm of auction theory but rather within the realm of decision models. The auction theory is predicated on the assumption of asymmetric information; the auctioneer (the public owner) must not know some parameters of the firm's (contractor) cost function. However, symmetric information is common in Japanese bidding practice. The ceiling price is interpreted as the 'standard price.' Workers' wages, materials prices, and productivity of each unit of work are surveyed nationally, and their mean values are obtained by type of construction, region, and season. These values are accumulated to form the ceiling price of each project. This study aims to obtain the optimum values of β and γ and analyse the characteristics of this model. This necessitates information on production costs, a common requirement in Japanese public works.

Here, the following assumptions are made:

1. The assumption of the public owner's objective function (O_1). It should be noted that ε_j, the company/engineer evaluation points of company *j*, are an important component of the CEM in Japan. All companies make their best efforts to increase these points. However, this study aims to analyse and discuss how innovative technologies (*t*) should be evaluated. Thus, as explained in the next section, we separate the evaluation of introduced technologies (*t*) and the company/engineer (ε_j) and treat the latter as random variables.
2. The assumption of one innovative company and multiple conventional companies.
   Previous studies have investigated and concluded that construction companies are seldom interested in innovation (Qiu *et al.* 2019, Wang and Chen 2023). In practice, there is only one innovative construction company in one region.
3. The assumption regarding ε_j:
   (a) $\varepsilon_j (j = 1, \ldots, n)$ is a mutually independent random variable.
   (b) $\varepsilon_j (j = 1, \ldots, n)$ is a random variable with identical distribution.

(c) Each ε_j is subject to uniform distribution between e_u and e_v, in which $\Delta e = e_u - e_v$.

4. The assumption regarding β, γ, t and α

   $t = t(\beta, \gamma)$: Owner's policy of β, γ, affects the bidder's decision of *t*.

   $\alpha = \alpha(\beta, \gamma)$: Owner's policy of β, γ, affect bidder's decision of α.

   α: Profit of both innovative and conventional companies are positive.

5. The assumption regarding $C(t(\beta, \gamma))$. Production cost, $C(t(\beta, \gamma))$, is assumed to be independent of ε, the value of technical points of each bidder. In practice, there is a possibility that ε influences the production cost. This article focuses on how to evaluate innovative technologies. This assumption enables us to do a series of analyses and identify an appropriate method of innovative technologies, the effects of removing the substantial LB and the mathematical characteristics of the formulated model.

   $C(t(\beta, \gamma))$ is a twice differentiable function of t. Further, there are three possible cases (Figure 5).

   Case 1) $\frac{\partial^2 C(t)}{\partial t^2} > 0$, $\frac{\partial C(0)}{\partial t} < 0$, and there exists $\tilde{t}$ satisfying $\frac{\partial C(\tilde{t})}{\partial t} = 0$

   Case 2) $\frac{\partial^2 C(t)}{\partial t^2} > 0$, $\frac{\partial C(t)}{\partial t} \geq 0$, and $\frac{\partial C(0)}{\partial t} = 0$

   Case 3) $\frac{\partial^2 C(t)}{\partial t^2} \leq 0$, $\frac{\partial C(0)}{\partial t} \geq 0$, and $\frac{\partial C(t_A)}{\partial t} = 0$

   where $t_A = \tilde{t}$ satisfying $\frac{\partial C(\tilde{t})}{\partial t} = 0$

6. All conventional bidders are assumed to bid at the substantially LB. In practice, bidders with a high value of ε may bid higher than the substantially LB. Like the (5) assumption, this enables us to do the above-mentioned analyses and identifications. To remove the (5) and (6) assumptions is a future issue.

7. No 'lemons' problems are assumed to exist. No bidders excessively bid low by sacrificing other management components, such as reducing labour wages and compromising quality. Lopomo *et al.* (2023) analyse the effectiveness of Lola (low-ball lottery auction) under the assumption of the existence of lemons. Lola is similar to pure competitive bidding in Japan: a floor price and a reserve price are set to exclude bids that are too low and too high, and a lottery is used to break a tie. To deal with the lemons problem, this article takes a different approach. Productivity improvement is considered one of the most important national goals in the construction industry in Japan. In the optimisation theory, for a solution to be optimum, it must satisfy the optimality and feasibility conditions. This study aims to conduct the optimality analyses: whether two proposals of this study, the evaluation method of new technologies and removal of the substantial LB, generate cost-effectiveness. The feasibility analysis of the proposals, whether lemons problems can be prevented, is being conducted by another study by the authors.

Results

Equilibrium solution in scenario without LB

Necessary and sufficient conditions for t^* and α^* to maximise O_2

In this section, we prove the existence of a local maximum of O_2. We demonstrate that a solution that satisfies the necessary conditions for a local maximum satisfies its sufficient conditions. We first take partial derivatives of O_2 with respect to t and α, respectively.

$$\frac{\partial O_2}{\partial t} = -\frac{\alpha}{\Delta e}x^{n-1}\frac{\partial \delta}{\partial t} + \frac{\alpha}{\Delta e}\frac{\partial \delta}{\partial t} = \frac{\alpha}{\Delta e}\frac{\partial \delta}{\partial t}(1 - x^{n-1}) \quad (21)$$

$$\frac{\partial O_2}{\partial \alpha} = \frac{1}{n}(1 - x^n) + \frac{\alpha}{n}(-n)x^{n-1}\frac{\partial x}{\partial \alpha} + x + \alpha\frac{\partial x}{\partial \alpha}$$

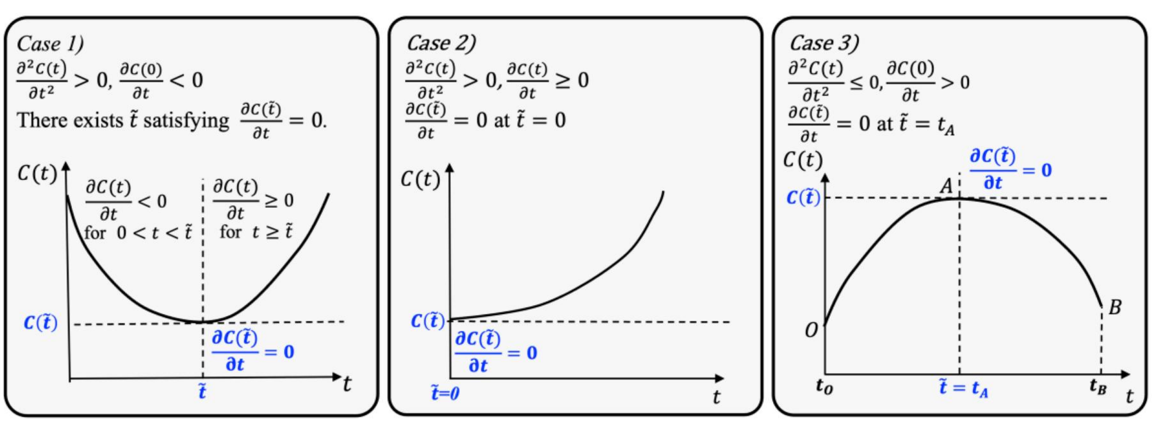

Figure 5. Three possible cases regarding $C(t)$.

$$= \frac{1}{n}(1 - x^n) + \frac{g}{\Delta e}(1 - x^{n-1}) + x \tag{22}$$

Suppose the following first-order conditions are satisfied.

$$\frac{\partial O_2}{\partial t} = \frac{\alpha}{\Delta e}\frac{\partial \delta}{\partial t}(1 - x^{n-1}) = 0 \tag{21′}$$

$$\frac{\partial O_2}{\partial \alpha} = \frac{1}{n}(1 - x^n) + \frac{g}{\Delta e}(1 - x^{n-1}) + x = 0 \tag{22′}$$

Then, the following lemma holds.

Lemma 1. When Equations (21′) and (22′) hold,

$$0 \le x < 1 \tag{11″}$$

$$\frac{\partial \delta}{\partial t} = \beta + h\gamma\frac{\partial C(t)}{\partial t} = 0 \tag{23}$$

$$\frac{\partial C(t)}{\partial t} = -\frac{\beta}{h\gamma} = \frac{\overline{P}(1 - s)\beta}{\gamma} \tag{23′}$$

are satisfied.

Proof: See Appendix B for details.

Equation (11″) represents the boundary of x. Equations (23) and (22′) are necessary conditions for a local minimum.

Three cases are analysed. For the first two cases, $\frac{\partial^2 C(t)}{\partial t^2} > 0$ is assumed; that is, $C(t)$ is a strictly convex function of t. The detailed conditions in the first two cases are given by (1) $\frac{\partial C(0)}{\partial t} < 0$, (2) $\frac{\partial C(t)}{\partial t} \ge 0$ and $\frac{\partial C(0)}{\partial t} = 0$. The first case is where cost is reduced as technology is introduced, and the latter is where cost is not immediately reduced with technology introduction (Figure 5). We will discuss Case 3 later. Then, the following theorem holds:

Theorem 2. t and α that satisfy necessary conditions (23) and (22′) form a local maximum of O_2.

Proof: See Appendix C for details.

Necessary and sufficient conditions for an equilibrium to minimise O_1

The following three processes generally obtain the equilibrium solution. The first step is to derive the analytical solution of decision variables t and α of the follower (i.e., the innovative company) as functions of β and γ from Equations (23) and (22′), respectively. The second step entails substituting them into Equation (1), which is the objective function of the leader (i.e., the public owner), and obtaining the values of β and γ by solving the simultaneous equations with the values of the first-order derivatives, with β and γ set to 0. The third step is to determine the values of t and α by substituting these values into the above-mentioned analytical solutions for t and α. This time, however, it is difficult to find the analytical solution of α from (22′).

Here, we take an alternative approach. The objective function of the owner, O_1, which is the awarded price of the innovative company, is given by

$$O_1 = b = C(t) + \alpha \tag{2′}$$

To derive the necessary and sufficient conditions for β and γ to be a local minimum of O_1, we must derive the first and the second derivatives of O_1 with respect to β and γ. Thus, the alternative approach consists of three steps. First, using Equations (23) and (22′), we derive the derivatives of implicit functions of t and α with respect to β and γ. Second, using the results of these derivatives, we obtain the necessary conditions for the local optimum using Khun-Tucker conditions. Third, we check sufficient conditions of solutions that satisfy necessary conditions by computing the determinant of the Hessian matrix.

Lemma 2. The first derivatives of O_1 with respect to β and γ are given by

$$\frac{dO_1}{d\beta} = \frac{dC(t)}{d\beta} + \frac{d\alpha}{d\beta} = \frac{1}{\frac{\partial^2 C(t)}{\partial t^2}}\frac{\beta}{(h\gamma)^2} + \frac{-1}{2h\gamma}\frac{J_1}{J_2}t > 0 \tag{24}$$

$$\frac{dO_1}{d\gamma} = \frac{dC(t)}{d\gamma} + \frac{d\alpha}{d\gamma}$$
$$= \frac{-1}{\frac{\partial^2 C(t)}{\partial t^2}}\frac{\beta^2}{h^2\gamma^3} + \frac{1}{\gamma}\left[\frac{1}{2}\left(\frac{J_1}{J_2}\right)\left(\overline{P}s - c(t)\right) - \alpha\right] \tag{25}$$

where

$$J_1 = 1 - x^{n-1} - \frac{(n-1)g}{\Delta e}x^{n-2} \tag{26}$$

$$J_2 = 1 - x^{n-1} - \frac{(n-1)g}{2\Delta e}x^{n-2} \tag{27}$$

Proof: See Appendix D for details.

Two observations are made here. First, from Equation (24), $\frac{dO_1}{d\beta} > 0 \neq 0$ holds. Second, for Equation (23) to be satisfied, the equations below hold:

$$\frac{\partial C(t)}{\partial t} \ge 0$$

That is,

$$t \ge \tilde{t} \tag{28}$$

where:

$$\frac{\partial C(\tilde{t})}{\partial t} = 0 \tag{29}$$

Thus, we must formulate the problem as a constrained minimisation problem to obtain the equilibrium solution. Then, the next theorem holds.

Theorem 3. The problem of obtaining the necessary conditions for a local minimum of O_1 is formulated as:

$$\begin{aligned} L &= O_1 + \lambda_1 v + \lambda_2 (t - \tilde{t}) \\ &= O_1 + \lambda_1 \left(\beta + h\gamma \frac{\partial C(t)}{\partial t} \right) + \lambda_2 (t - \tilde{t}) \end{aligned} \tag{30}$$

$$\frac{dL}{d\beta} = \frac{dC(t)}{d\beta} + \frac{d\alpha}{d\beta} + \lambda_2 \frac{dt}{d\beta} = \left(\frac{\partial C(t)}{\partial t} + \lambda_2 \right) \frac{dt}{d\beta} + \frac{d\alpha}{d\beta} = 0 \tag{31}$$

$$\frac{dL}{d\gamma} = \frac{dC(t)}{d\gamma} + \frac{d\alpha}{d\gamma} + \lambda_2 \frac{dt}{d\gamma} = \left(\frac{\partial C(t)}{\partial t} + \lambda_2 \right) \frac{dt}{d\gamma} + \frac{d\alpha}{d\gamma} = 0 \tag{32}$$

$$\frac{\partial L}{\partial t} = \frac{\partial C(t)}{\partial t} + \lambda_1 h\gamma \ \frac{\partial^2 C(t)}{\partial t^2} + \lambda_2 = 0 \tag{33}$$

$$\lambda_2 (t - \tilde{t}) = 0 \tag{34}$$

$$\lambda_1 \leq 0 \text{ and } \lambda_2 \leq 0 \tag{35}$$

The solution to the above formulation must satisfy the following conditions:

$$\frac{\partial C(t)}{\partial t} = 0 \tag{29$'$}$$

$$w = \frac{\partial O_2}{\partial \alpha} = \frac{1}{n}(1 - x^n) + \frac{g}{\Delta e}(1 - x^{n-1}) + x = 0 \tag{22$''$}$$

$$\beta = 0 \tag{36}$$

$$\frac{d\alpha}{d\gamma} = \frac{1}{\gamma} \left[\frac{1}{2} \left(\frac{J_1}{J_2} \right) \left(\bar{P}s - C(t) \right) - \alpha \right] = 0 \tag{37}$$

$$0 \leq x < 1 \tag{11$''$}$$

Proof: See Appendix E for details.

Equations (29′), (22″), (36), (37) and (11″) represent the necessary conditions for the local optimum. Next, we derive sufficient conditions. The next theorem holds.

Theorem 4. Sufficient conditions for a local minimum of O_1 are either given by:

$$u_1 \equiv \frac{1}{\gamma} \left(\frac{J_1}{J_2} \right) - 2 \left(\frac{J_1}{J_2} \right)'_{\gamma} \leq 0 \tag{38}$$

or when $u_1 = \frac{1}{\gamma}\left(\frac{J_1}{J_2}\right) - 2\left(\frac{J_1}{J_2}\right)'_{\gamma} > 0$,

$$u_2 \equiv \frac{\left(2 + \frac{J_1}{J_2}\right)\left(\frac{J_1}{J_2}\right)'_{\gamma}}{\left(\frac{J_1}{J_2}\right)\left\{\frac{1}{\gamma}\left(\frac{J_1}{J_2}\right) - 2\left(\frac{J_1}{J_2}\right)'_{\gamma}\right\}} = \frac{\left\{2\left(\frac{J_2}{J_1}\right) + 1\right\}\left(\frac{J_1}{J_2}\right)'_{\gamma}}{\left\{\frac{1}{\gamma}\left(\frac{J_1}{J_2}\right) - 2\left(\frac{J_1}{J_2}\right)'_{\gamma}\right\}} \geq \frac{\frac{\partial^2 C(t)}{\partial t^2} t^2}{\left\{\bar{P}s - C(t)\right\}} \tag{39}$$

Proof: See Appendix F for details.

The results of Equations (29′) and (36) mean that the technology that increases construction costs cannot become the equilibrium solution. In Case 2 (Figure 5), the innovative company cannot introduce new technology.

These results lead us to the discussion of Case 3 (Figure 5). It is sufficient to delineate whether t_A and t_B can constitute the equilibrium solution. First, we compare t_o and t_A. Without loss of generality, their comparison becomes possible by connecting points O and A with a convex cost function satisfying the conditions of Case 2. Thus, t_A can never constitute the equilibrium solution. Next, we compare t_o and t_B. This case is further subdivided into two cases: $C(t_o) > C(t_B)$ (Case 3-1) and $C(t_o) \leq C(t_B)$ (Case 3-2). Without loss of generality, by connecting points O and B with a convex cost function and adding other conditions of $\frac{\partial C(t_B)}{\partial t} = 0$ to Case 3-1 and $\frac{\partial C(t_O)}{\partial t} = 0$ to Case 3-2, the results of Case 1 apply to Case 3-1. Thus, t_B constitutes the equilibrium solution. The results of Case 2 apply to Case 3-2. Thus, $t = 0$ in Case 3-2 contradicts the setting that $t > 0$ for an innovation company.

Therefore, the necessary and sufficient conditions for the equilibrium solution to Scenario without LB satisfy Equations (29′), (22″), (36), (37) and (11″) and Equation (38) or (39). Equations (29′) and (22″) describe the necessary and sufficient conditions with respect to the follower—the innovative company's decision variables, t and α. Equations (36) and (37) describe the first order of necessary conditions with respect to the leader, the ordering party's decision variables, β and γ. Equations (11″) describe the minimum and supremum values of x. Equations (38) and (39) describe the sufficient conditions with respect to the ordering party's decision variables, β and γ.

Analysis of equilibrium in a scenario without LB

Equations (22″) and (37) need to be further analysed to determine the equilibrium solution. We derived equations for the equilibrium solution at the end of the last section. In the equilibrium solution, $\beta = 0$ and the t value can be easily obtained by solving $\frac{\partial C(t)}{\partial t} = 0$. In this section, we analyse the relationship between γ and α, determine the values of γ and α to satisfy necessary conditions, check their sufficiency conditions and identify their physical interpretations of γ and α.

Analysis of γ *and* α

In this section, we probe into the values of γ and α that satisfy the necessary conditions for an equilibrium. The following lemma holds.

Table 1. Summary of γ, α, x_γ, and x_α that satisfy Equations (29′), (22″), (36), (37) and (11″).

γ, α, and $\alpha\gamma$ and their equations		x's and their equations	
$\alpha\gamma = x_{\alpha\gamma}\{\bar{P}(1-s)\Delta e\}$	(40)	$x_{\alpha\gamma} = \frac{1-x^n+nx}{(1-x^{n-1})n}$	(41)
$\gamma = x_\gamma \frac{\bar{P}(1-s)\Delta e}{\bar{P}s-C(t^*)}$	(42)	$x_\gamma = \frac{1+2nx-(n+1)x^n}{(1-x^{n-1})n}$	(43)
$\alpha = x_\alpha\{\bar{P}s - C(t^*)\}$	(44)	$x_\alpha = \frac{1-x^n+nx}{1+2nx-(n+1)x^n}$	(45)

Lemma 3. The values of γ and α that satisfy Equations (29′), (22″), (36) and (11″), which are part of the necessary conditions for the equilibrium solution to Scenario without LB, are given in Table 1.

Proof: See Appendix G for details.

Determination of γ and α to satisfy necessary conditions

In this section, we determine the concrete values to satisfy γ and α to satisfy the necessary conditions.

Theorem 5. The necessary conditions in Scenario without LB, denoting $E_C(t^*, \alpha^*, \beta^*, \gamma^*)$, is:

$$E_C(t^*, \alpha^*, \beta^*, \gamma^*) = \begin{bmatrix} \frac{\partial C^{-1}(0)}{\partial t} \\ \left\{\frac{1-x^{*n}+nx^*}{1+2nx^*-(n+1)x^{*n}}\right\}(\bar{P}s - C(t^*)) = x^*_\alpha(\bar{P}s - C(t^*)) \\ 0 \\ \left\{\frac{1+2nx^*-(n+1)x^{*n}}{(1-x^{*n-1})n}\right\}\frac{\bar{P}(1-s)\Delta e}{(\bar{P}s - C(t^*))} = x^*_\gamma \frac{\bar{P}(1-s)\Delta e}{(\bar{P}s - C(t^*))} \end{bmatrix} \tag{46}$$

where $\frac{\partial C^{-1}}{\partial t}$ is the inverse function of $\frac{\partial C}{\partial t}$ and

$$f_1(x^*) = (n-1)^2 x^{*n} + nx^{*n-1} - 1 = 0 \tag{47}$$

x^*_α and x^*_γ are the values of x_α and x_γ when $x = x^*$.

Furthermore, x^* only depends on n, the number of bidders. Thus, x^*_α, x^*_γ and $x^*_{\alpha\gamma}$ depend on n.

Proof: See Appendix H for details.

Discussion of sufficient conditions

The following theorem holds to conduct a numerical analysis and discuss the satisfaction of sufficient conditions.

Theorem 6. For $4 \le n \le 20$, Equation (38), one of the sufficient conditions, holds for the solution that satisfies the necessary conditions given by Equation (46).

Proof: See Appendix I for details.

Physical interpretation and range of γ and α

This section identifies physical interpretation and the range of γ and α. The following lemma holds.

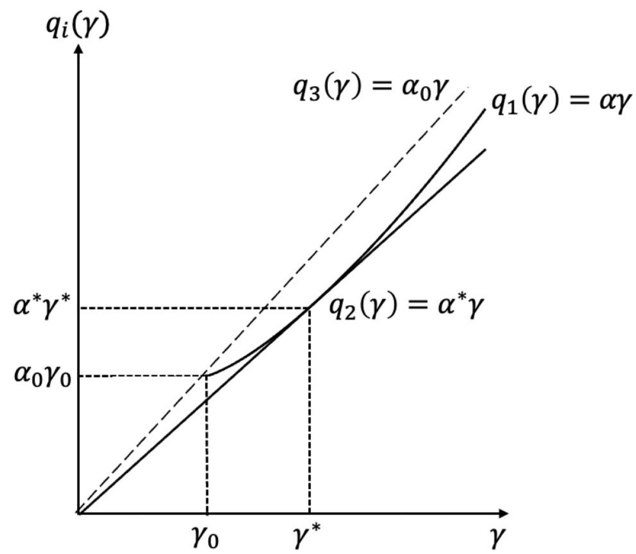

Figure 6. Relationship among γ and α with γ_o, α_o, γ^* and α^*

Lemma 4. For γ and α that satisfy Equations (29′), (22″), (36), (37) and (11″), $q_1(\gamma) = \alpha\gamma$ becomes a monotonously increasing function of γ and a convex function with respect to γ. Furthermore, for (α^*, γ^*) of the equilibrium solution, that additionally satisfies Equation (37), the slope of the tangent line of $q_2(\gamma)$ is a straight line with a slope α^* that passes through the origin and $(\gamma^*, \ \alpha^*\gamma^*)$.

Proof: See Appendix J for details.

Then, the theorem below holds.

Theorem 7. For γ and α that satisfy Equations (29′), (22″), (36) and (11″), the minimum value of γ, γ_o, and its supreme value, Supγ, are given by:

$$\gamma_o = \frac{\bar{P}(1-s)\Delta e}{\{\bar{P}s - C(t^*)\}n} \tag{48}$$

$$\text{Sup } \gamma = \infty \tag{49}$$

There is no maximum value of γ. Regarding the value of α, it takes the maximum value, α_o, when $\gamma = \gamma_o$; takes the minimum value, α^*, when $\gamma = \gamma^*$; and then approaches α_o as γ increases. α_o is given by:

$$\alpha_0 = \bar{P}s - C(t^*) \tag{50}$$

Proof: See Appendix K for details.

The relationship among γ and α with γ_o, α_o, γ^* and α^* is presented in Figure 6.

As seen in Figure 6, (α, γ) on $q_1(\gamma) = \alpha\gamma$ satisfies Equations (29′), (22″), and (36); $q_2(\gamma) = \alpha^*\gamma$ is a tangent line of $q_1(\gamma) = \alpha\gamma$, which goes through the origin; and α^* is the slope of this line and constitutes equilibrium. γ_0 is the minimum of γ, and α_0 is its associated value. Because *Sup*$\gamma = \infty$ holds from Equation

(49), there is no maximum value of γ. It should be noted that α is the slope of the line connecting a point on $q_1(\gamma) = \alpha\gamma$ and the origin. As γ increases from γ_0, α decreases from α_0, reaches the minimum value of α^*, starts increasing, and approaches α_0.

In practice, α^* is the minimum value the innovative company can accept given the public owner's strategy. It should be noted that α^* does not occur at γ_0, the minimum of γ. It means that a certain level of weight of γ becomes an incentive for the innovative company to lower α.

Equilibrium solution in the scenario with LB

For the innovative company (O_2), as shown in Figure 1, in scenario with LB, all bidders bid at the same price, that is, investigation price ($\underline{P} = \overline{P}s$); therefore, the profit of each company is determined by their construction cost ($C(t)$), profit $\alpha = \overline{P}s - C(t)$, that is a constant value. As shown in Equation (12), the winning probability of the innovative company is $Prob = \frac{1}{n}\left\{1 - \left(\frac{\beta t}{\Delta e}\right)^n\right\} + \frac{\beta t}{\Delta e}$. For the public owner ($O_1$), the bidding price ($\underline{P} = \overline{P}s$), which is the investigation price (Figure 1), is the cost. In this scenario, $O_1 = \overline{P}s$; that is, O_1 is constant. Therefore, the objective function of O_2 and O_1 is

$$O_2 = \alpha\left[\frac{1}{n}\left\{1 - \left(\frac{\beta t}{\Delta e}\right)^n\right\} + \frac{\beta t}{\Delta e}\right]$$

$$= \{\overline{P}s - C(t)\}\left[\frac{1}{n}\left\{1 - \left(\frac{\beta t}{\Delta e}\right)^n\right\} + \frac{\beta t}{\Delta e}\right] \quad (51)$$

$$O_1 = \overline{P}s \quad (52)$$

Thus, this scenario can be reduced to a game where two players simultaneously make decisions. Similarly to the Scenario without LB, we obtain the solution to satisfy the necessary first-order conditions for the local maximum and check its sufficiency conditions by checking its Hessian matrix. We take the first partial derivatives of O_2 with respect to β and t.

$$\frac{\partial O_2}{\partial \beta} = \frac{\{\overline{P}s - C(t)\}t}{\Delta e}\left\{1 - \left(\frac{\beta t}{\Delta e}\right)^{n-1}\right\} \quad (53)$$

$$\frac{\partial O_2}{\partial t} = \frac{\beta}{\Delta e}\left\{1 - \left(\frac{\beta t}{\Delta e}\right)^{n-1}\right\}(\overline{P}s - C(t)) - \left[\frac{1}{n}\left\{1 - \left(\frac{\beta t}{\Delta e}\right)^n\right\} + \frac{\beta t}{\Delta e}\right]\frac{\partial C(t)}{\partial t} \quad (54)$$

The first-order conditions are $\frac{\partial O_2}{\partial t} = 0$ and $\frac{\partial O_2}{\partial \beta} = 0$. Equations to represent these two conditions are given by:

$$\frac{\beta t}{\Delta e} = 1 \quad (55)$$

$$\frac{\partial C(t)}{\partial t} = 0 \quad (56)$$

It is useful to analyse this scenario using the four cases used in Scenario without LB: Case 1, Case 2, Case 3-1 and Case 3-2. It should be noted that all $C(t)$ are assumed to be strict convex functions of t. Furthermore, it is assumed that $\frac{\partial C(t)}{\partial t} \geq 0$ under Case 2 and Case 3-2 and $\frac{\partial C(t)}{\partial t} \leq 0$ under Case 1 and Case 3-1. Under Case 2 and Case 3-2, $t = 0$ cannot satisfy Condition (55). $t \neq 0$ can satisfy this condition but cannot satisfy Equation (56) because of $\frac{\partial C(t)}{\partial t} > 0$ for $t \neq 0$. Under these two cases, there is no optimum solution. Hence, our analysis can focus on Case 1 and Case 3-1 where $\frac{\partial C(t)}{\partial t} \leq 0$, and there exists t^* satisfying $\frac{\partial C(t^*)}{\partial t} = 0$. Under these cases, t^* satisfy the first-order conditions (55) and (56).

Next, we obtain the Hessian matrix by taking the second partial derivatives of O_2 with respect to β and t to prove β^* and t^* exist to maximise O_2 and analyse its determinants. Then, the next theorem holds. Those values associated with t^* that satisfy the first-order conditions are given as follows:

Theorem 8. The solutions, t^* and β^*, that satisfy the first-order conditions (55) and (56),

$$\frac{\beta^* t^*}{\Delta e} = 1 \quad (55')$$

$$\frac{\partial C(t^*)}{\partial t} = 0 \quad (56')$$

also satisfy the sufficient conditions. Thus, they become the local maximum.

Proof See Appendix L for details.

The equilibrium solution in Scenario with LB is summarised thus:

$$E^*(t^*, \alpha^*, \beta^*) = \left[\frac{\partial C^{-1}(0)}{\partial t}, \overline{P}s - C(t^*), \frac{\Delta e}{t^*}\right]^T \quad (57)$$

The probability of winning for the innovative company is

$$Prob = \frac{1}{n}\left\{1 - \left(\frac{\beta^* t^*}{\Delta e}\right)^n\right\} + \frac{\beta^* t^*}{\Delta e} = 1 \quad (58)$$

and the expected profit is given by

$$O_2 = Prob * \alpha^* = \overline{P}s - C(t^*) \quad (59)$$

Validity analysis with practical data

Equilibrium solution with practical data

The simulation was performed under the following settings:

$$C(t) = l(t) + d(t) = a_1(1 - a_2 t)^2 + \underline{l} \quad (60)$$

$$s = {}^{\underline{P}}/_{\overline{P}} \quad (7)$$

The case study data is based on the pilot project called Saga Bridge Substructure Construction in Japan. This project consists of the construction of two bridge piers. A rough terrain crane (conventional technology) is commonly used as standard technology in Japan, and a fast-erecting crane (new technology) is used on each bridge pier. The fast-erecting crane is considered a promising technology that can save labour for transportation within the site (MLIT 2021b). This pilot project aims to measure and compare the productivity of both methods (Seki *et al.* 2019). As the first and only research to measure and analyse on-site productivity in detail in Japan, the costs of labour, rebar work, formwork, and support and scaffolding are compared between conventional and new technology. In this project, $d(t)$ represents the equipment rental costs. The reduced cost coefficients (a_1 and a_2) and the remaining costs, such as materials and labour ($\underline{l}$), are based on the productivity analysis results of this pilot project in 2017 and 2018. The ceiling price of the pilot project is 100.58 million JPY. Furthermore, among construction projects ordered by national agencies, those with orders between 50 million and 500 million JPY account for 42% of the construction projects and 35% of the order amounts. Projects with a ceiling price of around 100 million JPY are prominent areas for local construction companies.

There are several types of CEM, depending on the technical difficulty and volume of the projects. Local construction companies focus on projects ordered with 'Construction Capability Evaluation Type II.' The ratio of the ceiling price to the substantial LB (s) and the difference in the maximum and minimum values of company/engineer evaluation points of all companies (Δe) are based on the analysed results of 3540 projects with Construction Capability Evaluation Type II ordered by MLIT in 2022 (National Institute for Land and Infrastructure Management 2022).

We set the maximum number of bidders to 20 since it hardly exceeds 20 in practice.[3] The parameters of this project are given in Table 2.

The process of deriving the equilibrium Scenarios is described with the assumptions of Table 2 and Equation (60).

- Equilibrium in scenario without LB

Equilibrium in scenario without LB is as follows:

$$E^*(t^*, \alpha^*, \beta^*, \gamma^*) = \begin{bmatrix} \dfrac{\partial C^{-1}(0)}{\partial t} \\ x_\alpha^*(\overline{P}s - C(t^*)) \\ 0 \\ x_\gamma^* \dfrac{\overline{P}(1-s)\Delta e}{(\overline{P}s - C(t^*))} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{2} \\ 7\dfrac{1}{2}\left\{\dfrac{1 - x^{*n} + nx^*}{1 + 2nx^* - (n+1)x^{*n}}\right\} \\ 0 \\ \dfrac{172}{75}\left\{\dfrac{1 + 2nx^* - (n+1)x^{*n}}{(1 - x^{*n-1})n}\right\} \end{bmatrix} \quad (61)$$

where

$$f_1(x^*) = (n-1)^2 x^{*n} + nx^{*n-1} - 1 = 0 \quad (47)$$

Therefore, the optimal awarded price (O_1^*) is as follows:

$$O_1^* = C(t^*) + \alpha^* = 82\frac{1}{2} + 7\frac{1}{2}x_\alpha^* = 82\frac{1}{2} + 7\frac{1}{2}\left\{\frac{1 - x^{*n} + nx^*}{1 + 2nx^* - (n+1)x^{*n}}\right\} \quad (62)$$

The expected profit of the innovative company (O_2^*) is as follows:

Table 2. Value of the parameters in case simulation.

Parameter		Value
a_1	The reduced cost coefficient	2.5 (million Japanese yen)
a_2	The coefficient associated with technology introduction	2
$\underline{l}$	The remaining costs, such as materials, labour	82.5 (million Japanese yen)
$\overline{P}$	The ceiling price	100 (million Japanese yen)
$\underline{P}$	The substantial lower bound	90 (million Japanese yen)
s	The ratio of the ceiling price to the substantial lower bound	0.9
Δe	The difference in the maximum and minimum values of company/engineer evaluation points of all companies	1.72
n	The number of bidders	2∼ 20

$$O_2^* = \left\{\frac{1}{n}\left(1 - x^{*^n}\right) + x^*\right\}\alpha^*$$
$$= 7\frac{1}{2}\left\{\frac{1}{n}\left(1 - x^{*^n}\right) + x^*\right\}\left\{\frac{1 - x^{*n} + nx^*}{1 + 2nx^* - (n+1)x^{*n}}\right\} \quad (63)$$

- Equilibrium in scenario with LB

The equilibrium solution in scenario with LB is as follows:

$$E^* = (t^*, \alpha^*, \beta^*) = \begin{bmatrix} \frac{\partial C^{-1}(0)}{\partial t} \\ \bar{P}s - C(t^*) \\ \frac{\Delta e}{t^*} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ 7\frac{1}{2} \\ \frac{86}{25} \end{bmatrix} \quad (64)$$

The awarded price is the investigation price ($\underline{P}$). Therefore, the awarded price (O_1^*) is

$$O_1^* = \underline{P} = 90 \quad (65)$$

and the expected profit is given by

$$O_2 = \left\{\bar{P}s - C(t^*)\right\}\left[\frac{1}{n}\left\{1 - \left(\frac{\beta^* t^*}{\Delta e}\right)^n\right\} + \frac{\beta^* t^*}{\Delta e}\right]$$
$$= \bar{P}s - C(t^*) = \alpha^* = 7\frac{1}{2} \quad (66)$$

In the scenario with LB, the innovative company's probability of winning is 1 (see Equation (58)), regardless of the number of bids issued. Therefore, the expected profit and the awarded price are constant. This situation is unrealistically advantageous to the innovative company. The optimum solution under scenario without LB depends on the number of bidders, n. A justified scenario is introduced to compare the performance of the solutions under the scenario with LB and the Scenario without LB on equal footing.

A justified scenario

We modify the settings in scenario with LB so that the innovative company obtains the expected profit from scenario without LB. In this situation, the expected profit for the innovative company decreases to O_2^* in the scenario without LB. The profit in bid (α) does not change in the Scenario with LB ($\bar{P}s - C(t^*)(58)$). This way, we decrease the winning probability in the Justified Scenario.

The next paragraph describes the derivation process of the results of Table 3.

In Table 3

$$x_\alpha^* = \frac{1 - x^{*n} + nx^*}{1 + 2nx^* - (n+1)x^{*n}} \quad (45')$$

Here, Equations (10′), (44′) and (45′) represent the winning probability, the profit in the bid and the value of x_α associated with x^*, respectively. We change β^* ($\beta^* = \frac{\Delta e}{t^*}$ (55′)) into $\tilde{\beta}$, and the related variables are as follows:

$$\tilde{y} = \frac{\tilde{\beta} t^*}{\Delta e} \quad (70)$$

The winning probability of the innovative company in the justified scenario, $\tilde{p}$, is represented with

$$\tilde{p} = \frac{1}{n}\left(1 - \tilde{y}^n\right) + \tilde{y} \quad (71)$$

The expected profit in the justified scenario, according to O_2 (51) in Scenario with LB under equilibrium condition, $\tilde{O}_2$, is given by

$$\tilde{O}_2 = \tilde{p}\left\{\bar{P}s - C(t^*)\right\} \quad (72)$$

The award price in the justified scenario with LB, $\tilde{O}_1$, is unchanged:

$$\tilde{O}_1 = C(t^*) + \left\{\bar{P}s - C(t^*)\right\} = \bar{P}s \quad (52)$$

Suppose the expected profit, $\tilde{O}_2$, is equal to O_2^* in scenario without LB, given by Equations (10′) and (44′):

$$\tilde{p}\left\{\bar{P}s - C(t^*)\right\} = prob * \alpha^*$$
$$= \left\{\frac{1}{n}\left(1 - x^{*^n}\right) + x^*\right\}x_\alpha^*\left\{\bar{P}s - C(t^*)\right\}$$
$$= \left\{\frac{1}{n}\left(1 - x^{*^n}\right) + x^*\right\}\left\{\frac{1 - x^{*n} + nx^*}{1 + 2nx^* - (n+1)x^{*n}}\right\}\left\{\bar{P}s - C(t^*)\right\} \quad (73)$$

Table 3. Difference in characteristics in the justified scenario and scenario without LB.

	Justified Scenario	Scenario without LB
Winning probability	$\left\{\frac{1}{n}\left(1 - x^{*^n}\right) + x^*\right\}x_\alpha^*$ (67)	$\frac{1}{n}\left(1 - x^{*^n}\right) + x^*$ (10′)
Profit in bid (α) (makeup)	$\bar{P}s - C(t^*)$(59)	$x_\alpha^*\left\{\bar{P}s - C(t^*)\right\}$(44′)
Awarded price (O_1)	$\bar{P}s$ (52)	$C(t^*) + x_\alpha^*\left\{\bar{P}s - C(t^*)\right\}$(68)
Difference in awarded price	$(1 - x_\alpha^*)\left\{\bar{P}s - C(t^*)\right\}$(69)	

$$\tilde{p} = \left\{\frac{1}{n}(1 - x^{*^n}) + x^*\right\} x^*_\alpha = \frac{\{1 - x^{*n} + nx^*\}^2}{n\{1 + 2nx^* - (n+1)x^{*n}\}} \tag{67}$$

The difference in the award price is obtained thus:

$$\tilde{O}_1 - O^*_1 = \overline{P}s - (C(t^*) + \alpha^*)$$
$$= \overline{P}s - \left[C(t^*) + x^*_\alpha\{\overline{P}s - C(t^*)\}\right]$$
$$= (1 - x^*_\alpha)\{\overline{P}s - C(t^*)\}$$
$$= \left\{1 - \frac{1 - x^{*n} + nx^*}{1 + 2nx^* - (n+1)x^{*n}}\right\}\{\overline{P}s - C(t^*)\} \tag{69}$$

Although the expected profit is the same under the justified scenario and scenario without LB, the composition of the probability of winning and the profit earned from the bid differs. In scenario without LB, the probability of winning is high, but the profit from the bid is set lower than that in justified scenario by a factor of $x^*_\alpha = \frac{1-x^{*n}+nx^*}{1+2nx^*-(n+1)x^{*n}}$. As a result, the award price is reduced by a factor of $\left(1 - x^*_\alpha = 1 - \left\{\frac{1-x^{*n}+nx^*}{1+2nx^*-(n+1)x^{*n}}\right\}\right)$. This difference constitutes opportunity costs.

There is one more noteworthy difference in both scenarios: variance of profit. Denoting variance of profit under Scenario without LB as $Var_{withoutLB}$, we obtain

$$Var_{withoutLB} = p^*(\alpha^* - \alpha^* p^*)^2 + (1 - p^*)\alpha^{*2}p^{*2}$$
$$= (1 - p^*)p^*\alpha^{*2} \tag{74}$$

where p^* and α^* are the winning probability and the profit at the optimum solution.

Similarly, Var $_j$, the variance of profit under Justified Scenario, is given by:

$$Var_j = (1 - \tilde{p})\tilde{p}\tilde{\alpha}^2 = \left(\frac{1 - x^*_\alpha p^*}{x^*_\alpha}\right)p^*\alpha^{*2} \tag{75}$$

where $\tilde{p}$ and $\tilde{\alpha}$ are the winning probability and the profit at the optimum solution.

Their difference is computed as:

$$Var_{withoutLB} - Var_j = \left\{(1 - p^*) - \left(\frac{1 - x^*_\alpha p^*}{x^*_\alpha}\right)\right\}p^*\alpha^{*2}$$
$$= \left(1 - \frac{1}{x^*_\alpha}\right)p^*\alpha^{*2} < 0 \tag{76}$$

since $0 < x^*_\alpha < 1$ from Equations (45′) and (47).

Thus, the profit variance under the scenario without LB is smaller than that of the justified scenario.

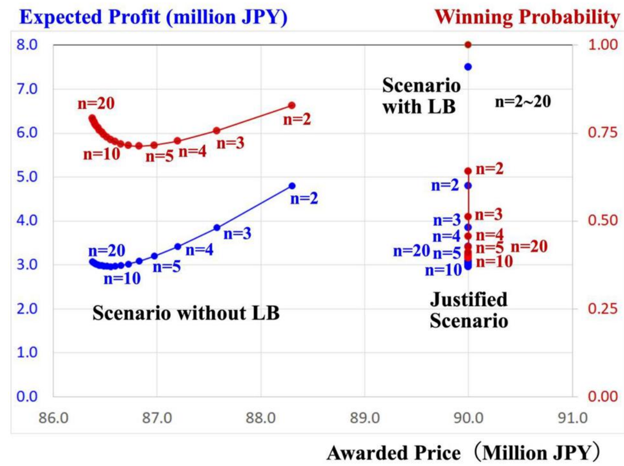

Figure 7. Equilibrium and optimum solutions in each scenario.

Simulation of scenario with LB, scenario without LB and justified scenario

The simulations in the three scenarios compare the expected profit and awarded price. Figure 7 illustrates the simulation results. In the scenario with LB, there is a lower bound. The winning probability is 1, and the expected profit and awarded price are constant regardless of the number of bidders, n. Scenario with LB is not a practical bidding method for introducing innovative technologies.

In the scenario without LB, the winning probability of the innovative company only depends on the number of bidders. Compared with the justified scenario, the scenario without LB has a lower awarded price, which benefits public owners. Despite the same expected profit, an innovative company has a higher winning probability ([0.713, 0.828]) than the justified scenario ([0.396, 0.641]). Thus, the innovative company may prefer the scenario without LB, where $\beta = 0$. However, in practice, public owners intend to balance innovative and conventional companies by lowering the winning probability of innovative companies. The justified scenario can be one example of the current multi-parameter approach to introducing innovative technology.

Discussion

A sequential decision-making problem in public bidding for the owner and the innovative company was successfully formulated as an optimisation problem using a Stackelberg game model, proving that its solution became a local optimum. Additionally, a method was developed to derive the optimal solution concisely. These advancements are regarded as significant academic contributions. The model's development and the analysis results highlight four key issues that emerge prominently.

First, the potential of the scenario without LB was identified through its comparison with the justified scenario. The solution in the scenario without LB has three key features: a lower awarded price, reduced profit variance, and a higher probability of winning. A lower awarded price provides a direct benefit to the public owner. Reduced variance of profit indicates that the innovative company faces less revenue uncertainty. A higher probability of winning suggests an increased likelihood of technology diffusion in society. The solution in the Scenario without LB may create three-way satisfaction among the public owner (buyer), the innovative company (seller), and society, which is also a traditional business philosophy in Japan. Thus, pursuing how to implement the scenario without LB is worthwhile.

Second, as depicted by Figure 6, too small and too large weight associated with a price point in the CEM (γ) increases the profit level the innovative company (α) secured and thus the awarded price. The existence of the optimum values of γ and α is expected to become a useful guideline for procurement officers when the Scenario without LB is implemented.

Third, the proposed method enables a strategic approach to technology diffusion. We obtained the result of the evaluation coefficient of new technologies, which is zero ($\beta = 0$) in the equilibrium solution of the Scenario without LB, showing that awarding a technology evaluation point ($\beta \neq 0$) to adopting new technology increases the award price. The result of the evaluation coefficient of new technologies, which is zero ($\beta = 0$), was derived from the minimisation of O_1. Thus, the introduction of different objective functions may give different equilibriums. For example, if a potential future cost reduction is introduced into O_1, we may see that the evaluation coefficient of new technologies is non-zero ($\beta \neq 0$). In the future, verifying the validity of the method presented here will be necessary by performing simulations that feature various scenarios.

If the above hypothesis is correct, the evaluation coefficient of new technologies (β) is set to a non-zero value to promote the more widespread use of the new technology, thereby reducing lease or depreciation costs. After the technology is regularly used, the public owner can set the evaluation coefficient of new technologies to zero, allowing the owner to enjoy cost reduction benefits. The proposed method enables a strategic approach to promote new technology and establish its market.

Fourth, the scenario with LB results highlights a common issue in public procurement: equitable distribution and innovation. Many Japanese public owners believe that public projects should be shared equally, as the funding for these projects comes from taxes. Another concern is the frequent occurrence of natural disasters, such as typhoons, floods, earthquakes, and tsunamis. Public owners require cooperation from local construction companies to restore infrastructure in a disaster. Thus, it seems complicated for them to justify the equilibrium solution, wherein an innovative company always wins a bid without visible cost reduction, while conventional enterprises cannot.

A justified scenario was created as an alternative to the scenario with LB to 'facilitate' the value of the Scenario without LB. Additionally, it functions as a link to the scenario without LB. One notable aspect is its lower winning probability for innovative companies compared to the scenario without LB, suggesting that conventional firms have more growth opportunities. As transitioning to the scenario without LB is the future path, it is prudent for conventional companies to adjust their strategies and prepare accordingly. A scenario like the Justified Scenario will give these companies the time necessary for transformation and sustainable growth.

Conclusion

The research presented a dynamic Stackelberg game model to formulate a multi-parameter bidding strategy. This model addresses a substantial LB in the Japanese construction industry, facilitating the integration of new technologies. The equilibrium solutions are categorised into two scenarios: Scenario with LB employs CEM, a modern multi-parameter approach, while Scenario without LB applies CEM without a substantial LB. In a case study conducted after the theoretical development, a Justified Scenario—an alternative to the Scenario with LB—was introduced, and its performance was compared with solutions from the other scenarios. The study found that the solutions under Scenario without LB, the lack of substantial LB may create three-way satisfaction among the public owner (buyer), the innovative company (seller), and society, and seem promising.

In contrast to the traditional model that relies on analytical solutions, this research employed the total derivatives of decision variables for both leaders and followers to formulate the equilibrium solution equations. This method offered a more profound insight into the problem. Additionally, this article introduces a strategy for creating multi-parameter bidding in construction innovation aimed at both public owners and contractors. The goal is to pinpoint optimal solutions that reduce costs for the public owner while maximising profits for the innovative contractor, ultimately improving the procurement design for construction innovations and highlighting the potential for a strategic approach to technology diffusion.

Limitations and future study

As an exploratory study, several limitations exist, primarily related to the assumptions made: independence between production cost and evaluation value for each bidder, the non-existence of a lemon, and the presence of one innovative company. Future studies should incorporate multiple innovative companies and introduce various objective functions for the public owner, such as considering expected cost reductions and analysing repeated games. In addition, a strategic model for disseminating new technology and establishing its market by adjusting a set of optimal parameter values is necessary for effective practice.

Notes

1. The ceiling prices, derived using a bottom-up approach, are interpreted as the 'standard price' and serve as the upper bound for the award price. Surveys are conducted all over Japan to collect necessary information, such as the average prices of materials, workers' wages, and productivity factors for each major type of construction work in each region. This information is then used to calculate the ceiling price.
2. Substantial LBs are determined by multiplying a certain ratio by the ceiling price. In estimating the cost of Japanese public works, the construction cost consists of four components: the direct construction costs, the temporary work costs, the field office expenses and the head office expenses. A reference model for determining the lower bound is multiplying different ratios by each cost component and adding them up. Public owners widely use this model.
3. In this analysis, the minimum ratio of an innovative company is set at 5%=1/20. One evidence for this is as follows. Authors have been promoting the fast-erecting crane for five years. Five companies with C rank and three with A rank have introduced this new technology in the Shikoku and Hokkaido areas. Four hundred sixty qualified companies with a C rank can participate in public works procured by the Shikoku Regional Bureau and the Ministry of Land, Infrastructure, Transport, and Tourism (MLIT). Thus, the ratio of innovators is 5/460 = 1.1%. The number of companies with A rank qualified for public works procured by Hokkaido Development Bureau, MLIT is 62. Thus, the ratio of innovators is 3/62 = 4.8%. In other regions, these ratios are even smaller.

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Authors' Contributions

Conceptualisation: R.W.; Methodology: R.W.; Validation: R.W., T.W. and M.S.; Formal analysis: R.W., T.W. and M.S.; Investigation: R.W. and T.W.; Resources: T.W.; Data curation: T.W. and M.S.; Writing – original draft: R.W., T.W. and M.S.; Writing – review & editing: M.S. and T.W.; Supervision: T.W. and M.S.; Project administration: M.S.; Funding acquisition: R.W.

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Data availability statement

The authors confirm that the data supporting the findings of this study are available within the article.

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Appendices

A. Proof of Theorem 1

The following conditions should be satisfied for the innovative company to exceed the evaluation value of conventional company *j*:

$$x_1 \geq x_j \tag{A1}$$

$$\beta t(\beta,\gamma) + \gamma \frac{\bar{P} - \{C(t(\beta,\gamma)) + \alpha(\beta,\gamma)\}}{\bar{P}(1-s)} + \varepsilon_1 \geq \gamma + \varepsilon_j$$

$$\varepsilon_j \leq \beta t(\beta,\gamma) + \gamma \frac{\bar{P}s - \{C(t(\beta,\gamma)) + \alpha(\beta,\gamma)\}}{\bar{P}(1-s)} + \varepsilon_1 \tag{A2}$$

Setting

$$CE(t,\alpha,\beta,\gamma) = \beta t(\beta,\gamma) + \gamma \frac{\bar{P}s - \{C(t(\beta,\gamma)) + \alpha(\beta,\gamma)\}}{\bar{P}(1-s)} \tag{A3),(13}$$

after which the probability that the evaluation value of the innovative company exceeds that of conventional company *j* is derived as

$$Pr[\varepsilon_j \leq \varepsilon_1 + CE(t,\alpha,\beta,\gamma)] \tag{A4}$$

With Assumption 3(a), the probability that the evaluation value of the innovative company will exceed those of all conventional companies is given by

$$Pr\left[\cap_{j=2}^{n}\varepsilon_j \leq \varepsilon_1 + CE(t,\alpha,\beta,\gamma)\right] = \prod_{j=2}^{n} Pr[\varepsilon_j \leq \varepsilon_1 + CE(t,\alpha,\beta,\gamma)] \tag{A5}$$

Therefore, using the total probability theorem and Assumption 3(b) yields the probability that the innovative company will win the bid of

$$\begin{aligned} Prob &= \sum_{All\ \varepsilon_1} Pr\left[\cap_{j=2}^{n}\varepsilon_j \leq \varepsilon_1 + CE(t,\alpha,\beta,\gamma)|\varepsilon = \varepsilon_1\right]Pr[\varepsilon = \varepsilon_1] \\ &= \sum_{All\ \varepsilon_1}\prod_{j=2}^{n} Pr[\varepsilon_j \leq \varepsilon_1 + CE(t,\alpha,\beta,\gamma)|\varepsilon = \varepsilon_1]Pr[\varepsilon = \varepsilon_1] \\ &= \int F_j(\varepsilon_1 + CE(t,\alpha,\beta,\gamma))^{n-1} f_1(\varepsilon_1)d\varepsilon_1 \end{aligned} \tag{A6}$$

With Assumption 3(c), we obtain:

$$Prob = \int_{e_u}^{e_v} F_j(\varepsilon_1 + CE(t,\alpha,\beta,\gamma))^{n-1} f_1(\varepsilon_1)d\varepsilon_1 \tag{A7}$$

$$= \int_{e_u}^{e_v - CE(t,\alpha,\beta,\gamma)} F_j(\varepsilon_1 + CE(t,\alpha,\beta,\gamma))^{n-1} f_1(\varepsilon_1)d\varepsilon_1 + \int_{e_v - CE(t,\alpha,\beta,\gamma)}^{e_v} F_j(\varepsilon_1 + CE(t,\alpha,\beta,\gamma))^{n-1} f_1(\varepsilon_1)d\varepsilon_1 \tag{A7'}$$

Here, the following relationships hold:

$$\begin{cases} f_1(\varepsilon_1)= f_1(\varepsilon_1 + CE(t,\alpha,\beta,\gamma)), e_u \le \varepsilon_1 \le e_v - CE(t,\alpha,\beta,\gamma) \\ F_j(\varepsilon_1 + CE(t,\alpha,\beta,\gamma)) = 1, e_v - CE(t,\alpha,\beta,\gamma) \le \varepsilon_1 \le e_v \end{cases} \tag{A8}$$

Setting

$$\varepsilon_1' = \varepsilon_1 + CE(t,\alpha,\beta,\gamma) \tag{A9}$$

Finally, the probability of scenario without LB that the innovative company will win the bid is

$$Prob = \int_{e_u+CE(t,\alpha,\beta,\gamma)}^{e_v} F_j(\varepsilon_1')^{n-1} f_1(\varepsilon_1') d\varepsilon_1' + \int_{e_v-CE(t,\alpha,\beta,\gamma)}^{e_v} f_1(\varepsilon_1) d\varepsilon_1$$
$$= \frac{1}{n}\left[1 - \left(\frac{e_u + CE(t,\alpha,\beta,\gamma) - e_u}{\Delta e}\right)^n\right] + 1 - \frac{e_v - CE(t,\alpha,\beta,\gamma) - e_u}{\Delta e}$$
$$= \frac{1}{n}\left[1 - \left(\frac{CE(t,\alpha,\beta,\gamma)}{\Delta e}\right)^n\right] + \frac{CE(t,\alpha,\beta,\gamma)}{\Delta e} \tag{10}$$

provided

$$0 \le \frac{CE(t,\alpha,\beta,\gamma)}{\Delta e} \le 1 \tag{11}$$

Because scenario with LB is represented by setting $\gamma = 0$ in Equation (A3), the probability of winning is:

$$Prob = \frac{1}{n}\left[1 - \left(\frac{\beta t(\beta)}{\Delta e}\right)^n\right] + \frac{\beta t(\beta)}{\Delta e} \tag{12}$$

Q.E.D.

B. Proof of Lemma 1

When $\frac{\partial O_2}{\partial t} = 0$, either $\frac{\partial \delta}{\partial t} = 0$ or $1 - x^{n-1} = 0$, that is, $x = 1$, is satisfied. Suppose that the equation below is satisfied:

$$x = 1 \tag{B1}$$

Substituting this into Equation (22) yields

$$\frac{\partial O_2}{\partial \alpha} = \frac{1}{n}\left(1 - (1)^n\right) + \frac{g}{\Delta e}\left(1 - (1)^{n-1}\right) + 1 = 1 \ne 0$$

Thus, Equation (B1) never holds, and

$$0 \le x < 1 \tag{11''}$$

is always satisfied. Accordingly, Equations (23) and (23′) must be satisfied.

Q.E.D.

C. Proof of Theorem 2

We take the second-order partial derivatives of O_2 with respect to t and α, respectively.

$$\frac{\partial^2 O_2}{\partial t^2} = \frac{\alpha}{\Delta e} h\gamma \frac{\partial^2 C(t)}{\partial t^2}\left(1 - x^{n-1}\right) \tag{C1}$$

$$\frac{\partial^2 O_2}{\partial \alpha^2} = \frac{1}{n}(-n)x^{n-1}\frac{\partial x}{\partial \alpha} + \frac{hr}{\Delta e}\left(1 - x^{n-1}\right) - \frac{g}{\Delta e}(n-1)x^{n-2}\frac{\partial x}{\partial \alpha} + \frac{\partial x}{\partial \alpha}$$
$$= \frac{2hr}{\Delta e}\left(1 - x^{n-1}\right) - (n-1)\alpha\left(\frac{hr}{\Delta e}\right)^2 x^{n-2} \tag{C2}$$

$$\frac{\partial^2 O_2}{\partial t \partial \alpha} = \frac{1}{\Delta e}\frac{\partial \delta}{\partial t}\left[\left\{1 - \left(\frac{\delta + g}{\Delta e}\right)^{n-1}\right\} - \frac{(n-1)g}{\Delta e}\left(\frac{\delta + g}{\Delta e}\right)^{n-2}\right]$$
$$= \frac{1}{\Delta e}\frac{\partial \delta}{\partial t}\left\{\left(1 - x^{n-1}\right) - \frac{(n-1)g}{\Delta e}x^{n-2}\right\} \tag{C3}$$

From Equations (11″), the following inequality equations hold.

$$\frac{\partial^2 O_2}{\partial t^2} < 0 \tag{C4}$$

$$\frac{\partial^2 O_2}{\partial \alpha^2} < 0 \tag{C5}$$

From Equation (23), we obtain

$$\frac{\partial^2 O_2}{\partial t \partial \alpha} = 0 \tag{C6}$$

The Hessian matrix of O_2 is given by

$$H_{O_2} = \begin{pmatrix} \frac{\partial^2 O_2}{\partial t^2} & \frac{\partial^2 O_2}{\partial \alpha \partial t} \\ \frac{\partial^2 O_2}{\partial \alpha \partial t} & \frac{\partial^2 O_2}{\partial \alpha^2} \end{pmatrix} \tag{C7}$$

The determinant of the Hessian Matrix is given by:

$$\det = \frac{\partial^2 O_2}{\partial t^2}\frac{\partial^2 O_2}{\partial \alpha^2} - \left(\frac{\partial^2 O_2}{\partial t \partial \alpha}\right)^2 = \frac{\partial^2 O_2}{\partial t^2}\frac{\partial^2 O_2}{\partial \alpha^2} > 0 \tag{C8}$$

From Equations (C4), (C5) and (C8), H_{O_2} is negative definite, which are sufficient conditions for local maximum. Hence, t and α that satisfy Equations Equations (23) and (22′) form a local maximum of O_2.

Q.E.D.

D. Proof of Proof of Lemma 2

For convenience of representation, we denote

$$v \equiv \frac{\partial \delta}{\partial t} = \beta + h\gamma\frac{\partial C(t)}{\partial t} = 0 \tag{D1),(23''}$$

$$w \equiv \frac{\partial O_2}{\partial \alpha} = \frac{1}{n}\left(1 - x^n\right) + \frac{g}{\Delta e}\left(1 - x^{n-1}\right) + x = 0 \tag{D2),(22''}$$

Since v and w are functions of t and α, respectively, which are implicit functions of β and γ, Equations (D1) and (D2) are rewritten as:

$$v = \frac{\partial \delta(t(\beta,\gamma),\beta,\gamma)}{\partial t} = \beta + h\gamma\frac{\partial C(t(\beta,\gamma))}{\partial t} = 0 \tag{D1'),(23'''}$$

$$w = \frac{\partial O_2}{\partial \alpha} = \frac{1}{n}\left(1 - x^n\right) + \frac{g}{\Delta e}\left(1 - x^{n-1}\right) + x$$
$$= \frac{1}{n}\left\{1 - x(t(\beta,\gamma),\alpha(\beta,\gamma),\beta,\gamma)^n\right\}$$
$$+ \frac{g(\alpha(\beta,\gamma),\gamma)}{\Delta e}\left\{1 - x(t(\beta,\gamma),\alpha(\beta,\gamma),\beta,\gamma)^{n-1}\right\}$$
$$+ x(t(\beta,\gamma),\alpha(\beta,\gamma),\beta,\gamma) = 0 \tag{D2'),(22'''}$$

By taking their direct derivatives with respect to β and γ, we obtain the following relationships.

$$\frac{dv}{d\beta} = \frac{\partial v}{\partial \beta} + \frac{\partial v}{\partial t}\frac{dt}{d\beta} = 0 \tag{D3}$$

$$\frac{dv}{d\gamma}=\frac{\partial v}{\partial \gamma}+\frac{\partial v}{\partial t}\frac{dt}{d\gamma}=0 \tag{D4}$$

$$\frac{dw}{d\beta}=\frac{\partial w}{\partial \beta}+\frac{\partial w}{\partial t}\frac{dt}{d\beta}+\frac{\partial w}{\partial \alpha}\frac{d\alpha}{d\beta}=0 \tag{D5}$$

$$\frac{dw}{d\gamma}=\frac{\partial w}{\partial \gamma}+\frac{\partial w}{\partial t}\frac{dt}{d\gamma}+\frac{\partial w}{\partial \alpha}\frac{d\alpha}{d\gamma}=0 \tag{D6}$$

Transforming Equations (D3) and (D4), $\frac{dt}{d\beta}$ and $\frac{dt}{d\gamma}$, the derivatives of implicit functions of t with respect to β and γ are obtained as

$$\frac{dv}{d\beta}=1+h\gamma\frac{\partial^2 C(t)}{\partial t^2}\frac{dt}{d\beta}=0 \tag{D3$'$}$$

$$\frac{dt}{d\beta}=-\frac{1}{\frac{\partial^2 C(t)}{\partial t^2}}\frac{1}{h\gamma}>0 \tag{D3$''$}$$

$$\frac{dv}{d\gamma}=h\frac{\partial C(t)}{\partial t}+h\gamma\frac{\partial^2 C(t)}{\partial t^2}\frac{dt}{d\gamma}=0 \tag{D4$'$}$$

$$\frac{dt}{d\gamma}=-\frac{1}{\gamma}\frac{\frac{\partial C(t)}{\partial t}}{\frac{\partial^2 C(t)}{\partial t^2}}=\frac{1}{\frac{\partial^2 C(t)}{\partial t^2}}\frac{\beta}{h\gamma^2}\leq 0 \tag{D4$''$}$$

The next step is to obtain the necessary conditions for the local minimum.

For simplification, we used the notation of w', x', δ' and g' to present their direct derivatives with respect to either β or γ. We denote $\delta'_\beta=\frac{d\delta}{d\beta}, g'_\beta=\frac{dg}{d\beta}, \delta'_\gamma=\frac{d\delta}{d\gamma}$ and $g'_\gamma=\frac{dg}{d\gamma}$ in detail. To further simplify the notation, we introduce the following notation:

$$J_1=1-x^{n-1}-\frac{(n-1)g}{\Delta e}x^{n-2} \tag{D7), (26}$$

$$J_2=1-x^{n-1}-\frac{(n-1)g}{2\Delta e}x^{n-2} \tag{D8), (27}$$

w' can be written as:

$$\begin{aligned} w' &= -x^{n-1}x'+\frac{1}{\Delta e}\left\{(1-x^{n-1})g'-(n-1)gx^{n-2}x'\right\}+x' \\ &=\frac{1}{\Delta e}\left[J_1(\delta'+g')+(1-x^{n-1})g'\right]=0 \end{aligned} \tag{D9}$$

It can also be written as:

$$\begin{aligned} w' &=\frac{1}{\Delta e}\left[\left\{1-x^{n-1}-\frac{(n-1)g}{\Delta e}x^{n-2}\right\}\delta'+2\left\{1-x^{n-1}-\frac{(n-1)g}{2\Delta e}x^{n-2}\right\}g'\right] \\ &=\frac{1}{\Delta e}\left[J_1\delta'+2J_2 g'\right]=0 \end{aligned} \tag{D10}$$

Taking the derivatives of w, δ and g with respect to β yields

$$\frac{dw}{d\beta}=\frac{1}{\Delta e}\left[J_1\delta'_\beta+2J_2 g'_\beta\right]=0 \tag{D11}$$

$$\begin{aligned} \delta'_\beta &=\frac{d\delta}{d\beta}=\frac{d\{\beta t+h\gamma(c(t)-\bar{P}s)\}}{d\beta}=t+\left(\beta+h\gamma\frac{\partial C(t)}{\partial t}\right)\frac{dt}{d\beta} \\ &=t>0 \end{aligned} \tag{D12}$$

$$g'_\beta=\frac{dg}{d\beta}=h\gamma\frac{d\alpha}{d\beta} \tag{D13}$$

It should be noted that the equations below hold.

$$J_1>0, J_2>0, \frac{J_1}{J_2}>0 \tag{D14}$$

Then, we obtain the following relationship:

$$\frac{d\alpha}{d\beta}=\frac{-1}{2h\gamma}\left(\frac{J_1}{J_2}\right)\delta'_\beta=\frac{-1}{2h\gamma}\left(\frac{J_1}{J_2}\right)t>0 \tag{D15}$$

Taking the derivative of w with respect to γ yields

$$\frac{dw}{d\gamma}=\frac{1}{\Delta e}\left[J_1\delta'_\gamma+2J_2 g'_\gamma\right]=0 \tag{D16}$$

$$g'_\gamma=-\frac{1}{2}\left(\frac{J_1}{J_2}\right)\delta'_\gamma \tag{D16$'$}$$

Each side of Equation (D16′) is developed as thus:

$$g'_\gamma=\frac{dg}{d\gamma}=h\left(\alpha+\gamma\frac{d\alpha}{d\gamma}\right) \tag{D17}$$

$$\begin{aligned} \delta'_\gamma &=\frac{d\delta}{d\gamma}=\frac{d\{\beta t+h\gamma(C(t)-\bar{P}s)\}}{d\gamma} \\ &=\beta\frac{dt}{d\gamma}+h(C(t)-\bar{P}s)+h\gamma\frac{\partial C(t)}{\partial t}\frac{dt}{d\gamma} \\ &=\left(\beta+h\gamma\frac{\partial C(t)}{\partial t}\right)\frac{dt}{d\gamma}+h(C(t)-\bar{P}s)=h(C(t)-\bar{P}s) \end{aligned} \tag{D18}$$

Subsequently, Equation (D16′) can be rewritten as:

$$\frac{d\alpha}{d\gamma}=\frac{1}{\gamma}\left[\frac{1}{2}\left(\frac{J_1}{J_2}\right)(\bar{P}s-C(t))-\alpha\right] \tag{D19}$$

As presented by Equation (2), O_1 is the sum of the cost and the profit.

$$O_1=b=C(t)+\alpha \tag{2$'$}$$

Thus, we obtain the following relationship:

$$\frac{dO_1}{d\beta}=\frac{\partial C(t)}{\partial t}\frac{dt}{d\beta}+\frac{d\alpha}{d\beta}=\frac{1}{\frac{\partial^2 C(t)}{\partial t^2}}\frac{\beta}{(h\gamma)^2}+\frac{-1}{2h\gamma}\frac{J_1}{J_2}t>0 \tag{24}$$

$$\frac{dO_1}{d\gamma}=\frac{\partial C(t)}{\partial t}\frac{dt}{d\gamma}+\frac{d\alpha}{d\gamma}=\frac{-1}{\frac{\partial^2 C(t)}{\partial t^2}}\frac{\beta^2}{h^2\gamma^3}+\frac{1}{\gamma}\left[\frac{1}{2}\left(\frac{J_1}{J_2}\right)(\bar{P}s-c(t))-\alpha\right] \tag{25}$$

Q.E.D.

E. Proof of Theorem 3

Necessary conditions for a local minimum of O_1 are given by solutions to the following constrained minimisation problem:

$$\begin{cases} Min\quad O_1(t(\beta,\gamma),\alpha(\beta,\gamma))=C(t(\beta,\gamma))+\alpha(\beta,\gamma)\quad (1) \\ S.T.: v=\beta+h\gamma\frac{\partial C(t(\beta,\gamma))}{\partial t}=0\quad (D1),(23''') \\ w=\frac{\partial O_2(t(\beta,\gamma),\alpha(\beta,\gamma),\beta,\gamma)}{\partial\alpha}=\frac{1}{n}(1-x(t(\beta,\gamma),\alpha(\beta,\gamma),\beta,\gamma)^n) \\ +\frac{g}{\Delta e}(1-x(t(\beta,\gamma),\alpha(\beta,\gamma),\beta,\gamma)^{n-1})+x(t(\beta,\gamma),\alpha(\beta,\gamma),\beta,\gamma)=0\ (D2)(22''') \\ t-\tilde{t}\geq 0\quad (28) \end{cases}$$

We introduce Lagrange multipliers λ_1, λ_2 and λ_3. A new objective function is written as:

$$L = O_1 + \lambda_1 v + \lambda_2(t - \tilde{t}) + \lambda_3 w$$
$$= O_1 + \lambda_1\left(\beta + h\gamma\frac{\partial C(t)}{\partial t}\right) + \lambda_2(t-\tilde{t}) + \lambda_3 w \tag{E1}$$

Then, necessary conditions for an optimal solution are given as follows.

$$\frac{dL}{d\beta} = \frac{dC(t)}{d\beta} + \frac{d\alpha}{d\beta} + \lambda_1\frac{dv}{d\beta} + \lambda_2\frac{dt}{d\beta} + \lambda_3\frac{dw}{d\beta} = 0 \tag{E2}$$
$$\frac{dL}{d\gamma} = \frac{dC(t)}{d\gamma} + \frac{d\alpha}{d\gamma} + \lambda_1\frac{dv}{d\gamma} + \lambda_2\frac{dt}{d\gamma} + \lambda_3\frac{dw}{d\gamma} = 0 \tag{E3}$$
$$\frac{\partial L}{\partial t} = \frac{\partial C(t)}{\partial t} + \lambda_1\frac{\partial v}{\partial t} + \lambda_2 + \lambda_3\frac{\partial w}{\partial t} = 0 \tag{E4}$$
$$\lambda_2(t-\tilde{t}) = 0 \tag{E5}$$
$$\lambda_1 \le 0, \lambda_2 \le 0, \text{ and } \lambda_3 \le 0 \tag{E6}$$

There is no boundary constraint for α like t described in Equation (28). $\frac{\partial L}{\partial \alpha}$ does not have to be included in the formulation for the necessary conditions.

$\frac{\partial w}{\partial t}$ is developed as

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} = \frac{\partial w}{\partial x}\left\{\frac{1}{\Delta e}\left(\frac{\partial \delta}{\partial t} + \frac{\partial g}{\partial t}\right)\right\} = 0 \tag{E7}$$

Using this equation as well as Equations (D3) to (D6), necessary conditions (E2), (E3) and (E4) are transformed into

$$\frac{dL}{d\beta} = \left(\frac{\partial C(t)}{\partial t} + \lambda_2\right)\frac{dt}{d\beta} + \frac{d\alpha}{d\beta} = 0 \tag{E2$'$}$$
$$\frac{dL}{d\gamma} = \left(\frac{\partial C(t)}{\partial t} + \lambda_2\right)\frac{dt}{d\gamma} + \frac{d\alpha}{d\gamma} = 0 \tag{E3$'$}$$
$$\frac{\partial L}{\partial t} = \frac{\partial C(t)}{\partial t} + \lambda_1 h\gamma\ \frac{\partial^2 C(t)}{\partial t^2} + \lambda_2 = 0 \tag{E4$'$}$$

Thus, it is unnecessary to include $\lambda_3 w$ in the new objective function. The objective function is reduced to:

$$L = O_1 + \lambda_1 v + \lambda_2(t-\tilde{t}) + \lambda_3 w$$
$$= O_1 + \lambda_1\left(\beta + h\gamma\frac{\partial C(t)}{\partial t}\right) + \lambda_2(t-\tilde{t}) \tag{30}$$

Substituting Equation (D3$''$) and Equation (D3$'''$)

$$\frac{dt}{d\beta} = -\frac{1}{\frac{\partial^2 C(t)}{\partial t^2}}\frac{1}{h\gamma} > 0 \ne 0 \tag{D3$'''$}$$

into Equation (E2$'$) yields

$$\frac{\partial C(t)}{\partial t} + \lambda_2 = -\frac{\frac{d\alpha}{d\beta}}{\frac{dt}{d\beta}} \tag{E8}$$

$$\lambda_2 = -\frac{\frac{d\alpha}{d\beta}}{\frac{dt}{d\beta}} - \frac{\partial C(t)}{\partial t} = \frac{-\frac{d\alpha}{d\beta}}{\frac{-1}{\frac{\partial^2 C(t)}{\partial t^2}h\gamma}} - \frac{\partial C(t)}{\partial t} = \frac{\partial^2 C(t)}{\partial t^2}\frac{d\alpha}{d\beta}h\gamma - \frac{\partial C(t)}{\partial t} \tag{E8$'$}$$

Suppose $t > \tilde{t}$; thus, $\frac{\partial C(t)}{\partial t} > 0$. Then, from Equation (E5), $\lambda_2 = 0$ holds.

In this case, however, the furthest right-hand side of Equation $(E8')$ is negative because $h < 0$, $\frac{\partial^2 C(t)}{\partial t^2} > 0$, $\frac{d\alpha}{d\beta} > 0$ and $\frac{\partial C(t)}{\partial t} > 0$. This is a contradiction.

Hence, we obtain

$$t = \tilde{t} \tag{E9}$$

and

$$\frac{\partial C(t)}{\partial t} = 0 \tag{29$'$}$$

From Equation (23), we obtain the following result:

$$\beta = 0 \tag{36}$$

Then, from Equations (D4$''$) and (E3$'$), the equations below hold.

$$\frac{dt}{d\gamma} = 0 \tag{E10}$$
$$\frac{d\alpha}{d\gamma} = 0 \tag{37}$$

Furthermore, from Equations (E8$'$) and (29$'$), the final value of λ_2 is given by:

$$\lambda_2 = \frac{\partial^2 C(t)}{\partial t^2}h\gamma\frac{d\alpha}{d\beta} < 0 \tag{E8$''$}$$

Substituting Equation (E8$''$) into Equation (E4$'$) yields

$$\lambda_1\frac{\partial^2 C(t)}{\partial t^2}h\gamma = -\lambda_2 = -\frac{\partial^2 C(t)}{\partial t^2}h\gamma\frac{d\alpha}{d\beta} \tag{E11}$$

Since $\frac{\partial^2 C(t)}{\partial t^2}h\gamma < 0 \ne 0$, we obtain the value of λ_1 as:

$$\lambda_1 = -\frac{d\alpha}{d\beta} < 0 \tag{E12}$$

Since both λ_1 and λ_2 are negative, a solution satisfying Equations (29$'$), (22$''$), (36), (37) and (D19$'$) satisfies the necessary conditions. Furthermore, Equations (11$''$) describe the minimum and supremum values of x.

Q.E.D.

F. Proof of Theorem 4

We take the second-order direct derivatives of O_1 with respect to β and γ, obtain the Hessian matrix of O_1 and compute its determinant. First, we analyse $C(t)$. Taking the first-order direct derivatives of $C(t)$ with β and γ, from Equations (23$'$), $(D3'')$ and $(D4'')$, we obtain

$$\frac{dC(t)}{d\beta} = \frac{\partial C(t)}{\partial t}\frac{dt}{d\beta} = \frac{\beta}{(h\gamma)^2\frac{\partial^2 C(t)}{\partial t^2}} \ge 0 \tag{F1}$$
$$\frac{dC(t)}{d\gamma} = \frac{\partial C(t)}{\partial t}\frac{dt}{d\gamma} = \frac{-\beta^2}{h^2\gamma^3\frac{\partial^2 C(t)}{\partial t^2}} \le 0 \tag{F2}$$

Then, the second-order direct derivatives of $C(t)$ with β and γ are given by

$$\frac{d^2 C(t)}{d\beta^2} = \frac{1}{(h\gamma)^2\frac{\partial^2 C(t)}{\partial t^2}} > 0 \tag{F3}$$
$$\frac{d^2 C(t)}{d\beta d\gamma} = \frac{-2\beta}{h^2\gamma^3\frac{\partial^2 C(t)}{\partial t^2}} = 0 \tag{F4}$$
$$\frac{d^2 C(t)}{d\gamma^2} = \frac{-3\beta^2}{h^2\gamma^4\frac{\partial^2 C(t)}{\partial t^2}} = 0 \tag{F5}$$

Next, we analyse α. Again, Equation (D15) is given by:

$$\frac{d\alpha}{d\beta} = \frac{-1}{2h\gamma}\left(\frac{J_1}{J_2}\right)t \tag{D15}$$

From Equations Equation (D16′) and (D18), the following equation holds.

$$g'_\gamma = \frac{dg}{d\gamma} = -\frac{1}{2}\left(\frac{J_1}{J_2}\right)\delta'_\gamma = \frac{h}{2}\left(\frac{J_1}{J_2}\right)\left(\overline{P}s - C(t)\right) \quad \text{(F6)}$$

Since $t > 0$,

$$\overline{P}s - C(t) > 0 \quad \text{(F7)}$$

Thus, the inequality below holds.

$$g'_\gamma = \frac{h}{2}\left(\frac{J_1}{J_2}\right)\left(\overline{P}s - C(t)\right) < 0 \quad \text{(F6′)}$$

Equation (F6′) can be transformed into

$$\frac{d(\alpha\gamma)}{d\gamma} = \frac{1}{2}\left(\frac{J_1}{J_2}\right)\left(\overline{P}s - C(t)\right) > 0 \quad \text{(F6″)}$$

For simplification, we denote $\left(\frac{J_1}{J_2}\right)'_\beta = \frac{d\left(\frac{J_1}{J_2}\right)}{d\beta}$, $\left(\frac{J_1}{J_2}\right)'_\gamma = \frac{d\left(\frac{J_1}{J_2}\right)}{d\gamma}$, $J_{1\beta}{}' = \frac{dJ_1}{d\beta}$, $J_{2\beta}{}' = \frac{dJ_2}{d\beta}$, $J_{1\gamma}{}' = \frac{dJ_1}{d\gamma}$, $J_{2\gamma}{}' = \frac{dJ_2}{d\gamma}$, $J_{3\beta} = J_{1\beta}{}'J_2 - J_{2\beta}{}'J_1$, and $J_{3\gamma} = J_{1\gamma}{}'J_2 - J_{2\gamma}{}'J_1$.

Directly differentiating Equation (D15) with respect to β and γ with substitution of Equations (D3″) and (D4″) yields

$$\frac{d^2\alpha}{d\beta^2} = \frac{-1}{2h\gamma}\left\{\left(\frac{J_1}{J_2}\right)'_\beta t + \left(\frac{J_1}{J_2}\right)\frac{dt}{d\beta}\right\}$$

$$= \frac{-1}{2h\gamma}\left\{\left(\frac{J_1}{J_2}\right)'_\beta t - \left(\frac{J_1}{J_2}\right)\frac{1}{h\gamma}\left(\frac{\partial^2 C(t)}{\partial t^2}\right)^{-1}\right\} \quad \text{(F8)}$$

$$\frac{d^2\alpha}{d\beta d\gamma} = \frac{-1}{2h}\left\{\left(\frac{J_1}{J_2}\right)'_\gamma \frac{t}{\gamma} + \left(\frac{J_1}{J_2}\right)\frac{\gamma\frac{dt}{d\gamma} - t}{\gamma^2}\right\}$$

$$= \frac{-1}{2h}\left\{\left(\frac{J_1}{J_2}\right)'_\gamma \frac{t}{\gamma} + \left(\frac{J_1}{J_2}\right)\frac{\left(\frac{\partial^2 C(t)}{\partial t^2}\right)^{-1}\frac{\beta}{h\gamma} - t}{\gamma^2}\right\} \quad \text{(F9)}$$

Directly differentiating Equation (F6′) with respect to γ yields

$$\frac{d^2(\alpha\gamma)}{d\gamma^2} = \gamma\frac{d^2\alpha}{d\gamma^2} + 2\frac{d\alpha}{d\gamma}$$

$$= \frac{1}{2}\left\{\left(\frac{J_1}{J_2}\right)'_\gamma\left(\overline{P}s - c(t)\right) - \left(\frac{J_1}{J_2}\right)\frac{\partial C(t)}{\partial t}\frac{dt}{d\gamma}\right\} \quad \text{(F10)}$$

Since $\beta = 0$, $\frac{\partial C(t)}{\partial t} = 0$, $\frac{d\alpha}{d\gamma}$, and $\frac{dt}{d\gamma} = 0$ hold, Equations (F9) and (F10) are further transformed into

$$\frac{d^2\alpha}{d\beta d\gamma} = \frac{-t}{2h\gamma}\left\{\left(\frac{J_1}{J_2}\right)'_\gamma - \left(\frac{J_1}{J_2}\right)\frac{1}{\gamma}\right\} \quad \text{(F11)}$$

$$\frac{d^2\alpha}{d\gamma^2} = \frac{1}{2\gamma}\left(\frac{J_1}{J_2}\right)'_\gamma\left(\overline{P}s - C(t)\right) \quad \text{(F12)}$$

We compute the values of $\left(\frac{J_1}{J_2}\right)'_\beta$ and $\left(\frac{J_1}{J_2}\right)'_\gamma$ and judge their signs. We first compute the value of $\left(\frac{J_1}{J_2}\right)'_\beta$. For $n = 2$, we obtain equations below.

$$J_1 = 1 - x^{n-1} - \frac{(n-1)g}{\Delta e}x^{n-2} = 1 - x - \frac{1}{\Delta e}g \quad \text{(D7), (26′)}$$

$$J_2 = 1 - x^{n-1} - \frac{(n-1)g}{2\Delta e}x^{n-2} = 1 - x - \frac{1}{2\Delta e}g \quad \text{(D8), (27′)}$$

$$J_{1\beta}{}' = -x'_\beta - \frac{1}{\Delta e}g'_\beta = -\frac{1}{\Delta e}\left\{\left(\delta'_\beta + g'_\beta\right) + g'_\beta\right\}$$

$$J_{2\beta}{}' = -x'_\beta - \frac{1}{2\Delta e}g'_\beta = -\frac{1}{\Delta e}\left\{\left(\delta'_\beta + g'_\beta\right) + \frac{1}{2}g'_\beta\right\}$$

$$J_{1\beta}{}'J_2 = -\frac{1}{\Delta e}\left\{\left(\delta'_\beta + g'_\beta\right) + g'_\beta\right\}\left\{1 - x - \frac{1}{2\Delta e}g\right\}$$

$$J_{2\beta}{}'J_1 = -\frac{1}{\Delta e}\left\{\left(\delta'_\beta + g'_\beta\right) + \frac{1}{2}g'_\beta\right\}\left\{1 - x - \frac{1}{\Delta e}g\right\}$$

$J_{3\beta}$ is given by

$$J_{3\beta} = J_{1\beta}{}'J_2 - J_{2\beta}{}'J_1 = -\frac{1}{2\Delta e}\left[\frac{1}{\Delta e}g\left(\delta'_\beta + g'_\beta\right) + (1-x)g'_\beta\right] \quad \text{(F13)}$$

Similarly, we obtain $J_{3\gamma}$ as

$$J_{3\gamma} = J_{1\gamma}{}'J_2 - J_{2\gamma}{}'J_1 = -\frac{1}{2\Delta e}\left[\frac{1}{\Delta e}g\left(\delta'_\gamma + g'_\gamma\right) + (1-x)g'_\gamma\right] \quad \text{(F14)}$$

For $n \geq 3$,

$$J_1 = 1 - x^{n-1} - \frac{(n-1)g}{\Delta e}x^{n-2} \quad \text{(D7), (26)}$$

$$J_2 = 1 - x^{n-1} - \frac{(n-1)g}{2\Delta e}x^{n-2} \quad \text{(D8), (27)}$$

Thus, we derive $J_{3\beta}$ as follows:

$$J_{1\beta}{}' = -\frac{(n-1)x^{n-3}}{\Delta e}\left[x\left(\delta'_\beta + g'_\beta\right) + \left\{xg'_\beta + (n-2)\frac{g}{\Delta e}\left(\delta'_\beta + g'_\beta\right)\right\}\right]$$

$$J_{2\beta}{}' = -\frac{(n-1)x^{n-3}}{\Delta e}\left[x\left(\delta'_\beta + g'_\beta\right) + \frac{1}{2}\left\{xg'_\beta + (n-2)\frac{g}{\Delta e}\left(\delta'_\beta + g'_\beta\right)\right\}\right]$$

$$J_{1\beta}{}'J_2 = -\frac{(n-1)x^{n-3}}{\Delta e}\left[x\left(\delta'_\beta + g'_\beta\right) + \left\{xg'_\beta + (n-2)\frac{g}{\Delta e}\left(\delta'_\beta + g'_\beta\right)\right\}\right]\left\{1 - x^{n-1} - \frac{(n-1)}{2\Delta e}gx^{n-2}\right\}$$

$$J_{2\beta}{}'J_1 = \frac{-(n-1)x^{n-3}}{\Delta e}\left[x\left(\delta'_\beta + g'_\beta\right) + \frac{1}{2}\left\{xg'_\beta + (n-2)\frac{g}{\Delta e}\left(\delta'_\beta + g'_\beta\right)\right\}\right]\left\{1 - x^{n-1} - \frac{(n-1)}{\Delta e}gx^{n-2}\right\}$$

Then $J_{3\beta}$ is given by:

$$J_{3\beta} = J_{1\beta}{}'J_2 - J_{2\beta}{}'J_1$$

$$= -\frac{(n-1)x^{n-3}}{\Delta e}\left[x\left(\delta'_\beta + g'_\beta\right)\frac{(n-1)}{2\Delta e}gx^{n-2} + \frac{1}{2}\left\{xg'_\beta + (n-2)\frac{g}{\Delta e}\left(\delta'_\beta + g'_\beta\right)\right\}(1 - x^{n-1})\right]$$

$$= \frac{-(n-1)x^{n-3}}{2\Delta e}\left[\left\{(n-2) + x^{n-1}\right\}\frac{g}{\Delta e}\left(\delta'_\beta + g'_\beta\right) + x(1 - x^{n-1})g'_\beta\right] \quad \text{(F15)}$$

Similarly, we compute $J_{3\gamma}$.

$$J_{1\gamma}{}' = \frac{-(n-1)x^{n-3}}{\Delta e}\left[x\left(\delta'_\gamma + g'_\gamma\right) + \left\{xg'_\gamma + (n-2)\frac{g}{\Delta e}\left(\delta'_\gamma + g'_\gamma\right)\right\}\right]$$

$$J_{2\gamma}{}' = \frac{-(n-1)x^{n-3}}{\Delta e}\left[x\left(\delta'_\gamma + g'_\gamma\right) + \frac{1}{2}\left\{xg'_\gamma + (n-2)\frac{g}{\Delta e}\left(\delta'_\gamma + g'_\gamma\right)\right\}\right]$$

$$J_{3\gamma} = J_{1\gamma}{}'J_2 - J_{2\gamma}{}'J_1$$
$$= \frac{-(n-1)x^{n-3}}{2\Delta e}\left[\left\{(n-2) + x^{n-1}\right\}\frac{g}{\Delta e}\left(\delta'_\gamma + g'_\gamma\right) + x(1-x^{n-1})g'_\gamma\right] \tag{F16}$$

We check the signs of $\delta'_\gamma + g'_\gamma$ to judge the signs of $J_{3\beta}$ and $J_{3\gamma}$. Transformation of Equation (D9) yields

$$\delta' + g' = -\frac{(1-x^{n-1})}{J_1}g' \tag{F17}$$

Using Equations (D13), (D14) and (D15), we obtain

$$g'_\beta = \frac{dg}{d\beta} = h\gamma\frac{d\alpha}{d\beta} = -\frac{1}{2}\left(\frac{J_1}{J_2}\right)\delta'_\beta = -\frac{1}{2}\left(\frac{J_1}{J_2}\right)t < 0 \tag{F18}$$

$$\delta'_\beta + g'_\beta = -\frac{(1-x^{n-1})}{J_1}g'_\beta > 0 \tag{F19}$$

Using Equation (F6′), we obtain

$$\delta'_\gamma + g'_\gamma = \frac{-(1-x^{n-1})}{J_1}g'_\gamma > 0 \tag{F20}$$

From Equations (F13), (F15), (F18) and (F19), the following inequalities are satisfied.

$$J_{3\beta} = J_{1\beta}{}'J_2 - J_{2\beta}{}'J_1 > 0 \tag{F21}$$

$$\left(\frac{J_1}{J_2}\right)'_\beta = \frac{J_{3\beta}}{J_2{}^2} > 0 \tag{F22}$$

Similarly, from Equations (F14), (F16), (F20) and (F21), we obtain

$$J_{3\gamma} = J_{1\gamma}{}'J_2 - J_{2\gamma}{}'J_1 > 0 \tag{F23}$$

$$\left(\frac{J_1}{J_2}\right)'_\gamma = \frac{J_{3\gamma}}{J_2{}^2} > 0 \tag{F24}$$

Using Equations (D14), (F8), (F22), (F12) and F(24), we can derive the following results:

$$\frac{d^2\alpha}{d\beta^2} = \frac{-1}{2h\gamma}\left\{\left(\frac{J_1}{J_2}\right)'_\beta t - \left(\frac{J_1}{J_2}\right)\frac{1}{h\gamma}\left(\frac{\partial^2 C(t)}{\partial t^2}\right)^{-1}\right\} > 0 \tag{F8′}$$

$$\frac{d^2\alpha}{d\gamma^2} = \frac{1}{2\gamma}\left(\frac{J_1}{J_2}\right)'_\gamma \left(\bar{P}s - c(t)\right) > 0 \tag{F12′}$$

From Equations (F3), (F8′), (F4), (F11), (F5) and (F12′), the results of the direct second-order derivatives of O_1 with respect to β and γ are summarised as follows:

$$\frac{d^2O_1}{d\beta^2} = \frac{d^2C}{d\beta^2} + \frac{d^2\alpha}{d\beta^2}$$
$$= -\frac{1}{2h\gamma}\left\{\left(\frac{J_1}{J_2}\right)'_\beta t - \left(2 + \frac{J_1}{J_2}\right)\frac{1}{h\gamma}\left(\frac{\partial^2 C(t)}{\partial t^2}\right)^{-1}\right\} > 0 \tag{F25}$$

$$\frac{d^2O_1}{d\beta d\gamma} = \frac{d^2C}{d\beta d\gamma} + \frac{d^2\alpha}{d\beta d\gamma} = \frac{-t}{2h\gamma}\left\{\left(\frac{J_1}{J_2}\right)'_\gamma - \left(\frac{J_1}{J_2}\right)\frac{1}{\gamma}\right\} \tag{F26}$$

$$\frac{d^2O_1}{d\gamma^2} = \frac{d^2C}{d\gamma^2} + \frac{d^2\alpha}{d\gamma^2} = \frac{1}{2\gamma}\left\{\left(\frac{J_1}{J_2}\right)'_\gamma \left(\bar{P}s - C(t)\right)\right\} > 0 \tag{F27}$$

The Hessian matrix of O_1 is given by:

$$H_{O_1} = \begin{pmatrix} \dfrac{d^2O_1}{d\beta^2} & \dfrac{d^2O_1}{d\beta d\gamma} \\ \dfrac{d^2O_1}{d\beta d\gamma} & \dfrac{d^2O_1}{d\gamma^2} \end{pmatrix} \tag{F28}$$

The determinant of the Hessian Matrix is given by:

$$\det = \frac{d^2O_1}{d\beta^2}\frac{d^2O_1}{d\gamma^2} - \left(\frac{d^2O_1}{d\beta d\gamma}\right)^2 \tag{F29}$$

Before we compute the determinant, we analyse the relationship between $\left(\frac{J_1}{J_2}\right)'_\beta$ and $\left(\frac{J_1}{J_2}\right)'_\gamma$. Differentiating Equation (D19) with respect to β and substituting Equation (D15) into it, the following equation holds.

$$\gamma\frac{d^2\alpha}{d\gamma d\beta} = \frac{1}{2}\left(\frac{J_1}{J_2}\right)'_\beta \left(\bar{P}s - c(t)\right) - \frac{d\alpha}{d\beta}$$
$$= \frac{1}{2}\left(\frac{J_1}{J_2}\right)'_\beta \left(\bar{P}s - c(t)\right) + \frac{1}{2h\gamma}\frac{J_1}{J_2}t \tag{F30}$$

Equating Equations (F11) and (F30) yields

$$\gamma\frac{d^2\alpha}{d\beta d\gamma} = \frac{-t}{2h}\left\{\left(\frac{J_1}{J_2}\right)'_\gamma - \left(\frac{J_1}{J_2}\right)\frac{1}{\gamma}\right\}$$
$$= \frac{1}{2}\left(\frac{J_1}{J_2}\right)'_\beta \left(\bar{P}s - c(t)\right) + \frac{1}{2h\gamma}\frac{J_1}{J_2}t$$
$$\left(\frac{J_1}{J_2}\right)'_\gamma = \left(\frac{-h}{t}\right)\left(\frac{J_1}{J_2}\right)'_\beta \left(\bar{P}s - c(t)\right) \tag{F31}$$

Substituting Equation (F31), the first term of the right-hand side of Equation (F29) is transformed into:

$$\frac{d^2O_1}{d\beta^2}\frac{d^2O_1}{d\gamma^2} = -\frac{1}{2h\gamma}\left\{\left(\frac{J_1}{J_2}\right)'_\beta t - \left(2 + \frac{J_1}{J_2}\right)\frac{1}{h\gamma}\left(\frac{\partial^2 C(t)}{\partial t^2}\right)^{-1}\right\}\frac{d^2O_1}{d\gamma^2}$$
$$= -\frac{1}{2h\gamma}\left(\frac{J_1}{J_2}\right)'_\beta t\frac{d^2O_1}{d\gamma^2} + \frac{1}{2h^2\gamma^2}\left\{\left(2 + \frac{J_1}{J_2}\right)\left(\frac{\partial^2 C(t)}{\partial t^2}\right)^{-1}\right\}\frac{d^2O_1}{d\gamma^2}$$
$$= \frac{t^2}{4h^2\gamma^2}\left\{\left(\frac{J_1}{J_2}\right)'_\gamma\right\}^2 + \frac{1}{2h^2\gamma^2}\left\{\left(2 + \frac{J_1}{J_2}\right)\left(\frac{\partial^2 C(t)}{\partial t^2}\right)^{-1}\right\}\frac{d^2O_1}{d\gamma^2}$$

Its second term is developed into:

$$\left(\frac{d^2O_1}{d\beta d\gamma}\right)^2 = \frac{t^2}{4h^2\gamma^2}\left[\left\{\left(\frac{J_1}{J_2}\right)'_\gamma\right\}^2 - \frac{2}{\gamma}\left(\frac{J_1}{J_2}\right)'_\gamma\left(\frac{J_1}{J_2}\right) + \frac{1}{\gamma^2}\left\{\left(\frac{J_1}{J_2}\right)\right\}^2\right]$$

Thus,

$$\det = \frac{1}{2h^2\gamma^2}\left\{\left(2 + \frac{J_1}{J_2}\right)\left(\frac{\partial^2 C(t)}{\partial t^2}\right)^{-1}\right\}\frac{d^2O_1}{d\gamma^2}$$
$$- \frac{t^2}{4h^2\gamma^3}\left(\frac{J_1}{J_2}\right)\left\{-2\left(\frac{J_1}{J_2}\right)'_\gamma + \frac{1}{\gamma}\left(\frac{J_1}{J_2}\right)\right\} \tag{F32}$$

For the determinant to be nonpositive, the following inequality must hold.

$$\left(\frac{\partial^2 C(t)}{\partial t^2}\right)^{-1} \geq \frac{t^2\left(\frac{J_1}{J_2}\right)\left\{-2\left(\frac{J_1}{J_2}\right)'_\gamma + \frac{1}{\gamma}\left(\frac{J_1}{J_2}\right)\right\}}{2\gamma\left(2+\frac{J_1}{J_2}\right)\frac{d^2O_1}{d\gamma^2}} = \frac{t^2\left(\frac{J_1}{J_2}\right)\left\{\frac{1}{\gamma}\left(\frac{J_1}{J_2}\right) - 2\left(\frac{J_1}{J_2}\right)'_\gamma\right\}}{\left(2+\frac{J_1}{J_2}\right)\left(\frac{J_1}{J_2}\right)'_\gamma\{\bar{P}s - C(t)\}} \quad \text{(F33)}$$

When

$$u_1 = \frac{1}{\gamma}\left(\frac{J_1}{J_2}\right) - 2\left(\frac{J_1}{J_2}\right)'_\gamma \leq 0 \quad (38)$$

holds, Equation (F33) is always satisfied, and the Hessian matrix becomes positive definite. In the case of $u_1 = \frac{1}{\gamma}\left(\frac{J_1}{J_2}\right) - 2\left(\frac{J_1}{J_2}\right)'_\gamma > 0$, this condition is equivalent to

$$\frac{\partial^2 C(t)}{\partial t^2} \leq \frac{\left(2+\frac{J_1}{J_2}\right)\left(\frac{J_1}{J_2}\right)'_\gamma\{\bar{P}s - C(t)\}}{t^2\left(\frac{J_1}{J_2}\right)\left\{\frac{1}{\gamma}\left(\frac{J_1}{J_2}\right) - 2\left(\frac{J_1}{J_2}\right)'_\gamma\right\}} \quad \text{(F33}')$$

If the following condition

$$u_2 = \frac{\left(2+\frac{J_1}{J_2}\right)\left(\frac{J_1}{J_2}\right)'_\gamma}{\left(\frac{J_1}{J_2}\right)\left\{\frac{1}{\gamma}\left(\frac{J_1}{J_2}\right) - 2\left(\frac{J_1}{J_2}\right)'_\gamma\right\}} = \frac{\left\{2\left(\frac{J_2}{J_1}\right)+1\right\}\left(\frac{J_1}{J_2}\right)'_\gamma}{\left\{\frac{1}{\gamma}\left(\frac{J_1}{J_2}\right) - 2\left(\frac{J_1}{J_2}\right)'_\gamma\right\}} \geq \frac{\frac{\partial^2 C(t)}{\partial t^2}t^2}{\{\bar{P}s - C(t)\}} \quad (39)$$

is satisfied, the Hessian matrix becomes positive semi-definite.

Thus, Equations (38) and (39) become sufficient conditions for a local minimum of O_1.

Q.E.D.

G. Proof of Lemma 3

Equation (20) is rewritten as:

$$\delta + g = x\Delta e \quad (20')$$

First, we treat x as a parameter, taking $0 \leq x < 1$ (11″).

$$\text{FromEquation}\ \frac{\partial O_2}{\partial \alpha} = \frac{1}{n}(1-x^n) + \frac{g}{\Delta e}(1-x^{n-1}) + x = 0(22') :$$

$$g = \Delta e\left\{\frac{1-x^n+nx}{(x^{n-1}-1)n}\right\} \quad \text{(G1)}$$

From Equation $h = \frac{-1}{\bar{P}(1-s)}$ (15), and $g = h\alpha\gamma(18')$:

$$\alpha\gamma = \left\{\frac{1-x^n+nx}{(1-x^{n-1})n}\right\}\{\bar{P}(1-s)\Delta e\} = x_{\alpha\gamma}\{\bar{P}(1-s)\Delta e\} \quad \text{(G2), (40)}$$

where

$$x_{\alpha\gamma} = \frac{1-x^n+nx}{(1-x^{n-1})n} \quad \text{(G3), (41)}$$

Given that both the numerator and the denominator of $x_{\alpha\gamma}$ take positive values, $\alpha\gamma$ becomes positive. Equations (20′) and (G2)and (40) yield

$$\delta = x\Delta e + \frac{\Delta e(1-x^n+nx)}{(1-x^{n-1})n} = \frac{\Delta e\{1+2nx-(n+1)x^n\}}{(1-x^{n-1})n} \quad \text{(G4)}$$

With Equation $\delta = \beta t(\beta,\gamma) + \gamma\frac{\bar{P}s - C(t(\beta,\gamma))}{\bar{P}(1-s)}$ (14),

$$\beta t + \gamma\frac{\bar{P}s - C(t)}{\bar{P}(1-s)} = \frac{\Delta e\{1+2nx-(n+1)x^n\}}{(1-x^{n-1})n}$$

$$\gamma\frac{\bar{P}s - C(t)}{\bar{P}(1-s)} = \frac{\Delta e\{1+2nx-(n+1)x^n\}}{(1-x^{n-1})n} - \beta t \quad \text{(G5)}$$

In the equilibrium, $\beta^* = 0$ and $t = t^*$ minimises $C(t)$. Thus, Equation (G5) becomes

$$\gamma = \left\{\frac{1+2nx-(n+1)x^n}{(1-x^{n-1})n}\right\}\frac{\bar{P}(1-s)\Delta e}{\{\bar{P}s - C(t^*)\}} = x_\gamma\frac{\bar{P}(1-s)\Delta e}{\{\bar{P}s - C(t^*)\}} \quad \text{(G6), (42)}$$

where

$$x_\gamma = \frac{1+2nx-(n+1)x^n}{(1-x^{n-1})n} \quad \text{(G7), (43)}$$

From Equations (40) and (41)((G2)and (G3)),

$$\alpha = \left\{\frac{1-x^n+nx}{1+2nx-(n+1)x^n}\right\}\{\bar{P}s - C(t^*)\} = x_\alpha\{\bar{P}s - C(t^*)\} \quad \text{(G8), (44)}$$

where

$$x_\alpha = \frac{1-x^n+nx}{1+2nx-(n+1)x^n} \quad \text{(G9), (45)}$$

$1+2nx-(n+1)x^n$, the numerator of x_γ and the denominator of x_α can be transformed:

$$1+2nx-(n+1)x^n = (1-x^n) + nx(1-x^{n-1}) + nx > 0(G10)$$

It is noted that, thus, γ and α take positive values.

Thus, the results of $x_\gamma, \gamma, x_\alpha, \alpha, x_{\alpha\gamma}$ and $\alpha\gamma$ are summarised in Table 1.

Q.E.D.

H. Proof of Theorem 5

Taking the first partial derivative of α (44) with respect to x, we obtain

$$\frac{\partial\alpha}{\partial x} = \frac{\{\bar{P}s - C(t^*)\}n}{\{1+2nx-(n+1)x^n\}^2}\Big[(1-x^{n-1})\{1+2nx-(n+1)x^n\} - \{2-(n+1)x^{n-1}\}(1-x^n+nx)\Big]$$

Setting

$$\begin{aligned} K &= (1-x^{n-1})\{1+2nx-(n+1)x^n\} - \{2-(n+1)x^{n-1}\}(1-x^n+nx) \\ &= -1 + nx^{n-1} + \{n(n+1)+2-2n-(n+1)\}x^n \\ &= (n-1)^2x^n + nx^{n-1} - 1 \end{aligned}$$

Hence,

$$\frac{\partial\alpha}{\partial x} = \frac{nf_1(x)}{\{1+2nx-(n+1)x^n\}^2}\{\bar{P}s - C(t^*)\} \quad \text{(H1)}$$

where

$$f_1(x) = (n-1)^2 x^n + nx^{n-1} - 1 \tag{H2}$$

Here, we examine the characteristics of $f_1(x)$.

$$f_1(0) = -1, \lim_{x \to 1} f_1(x) = (n-1)^2 + n - 1 = n(n-1) > 0 \tag{H3}$$

$$\frac{\partial f_1(x)}{\partial x} = n(n-1)^2 x^{n-1} + n(n-1)x^{n-2}$$
$$= n(n-1)x^{n-2}\{(n-1)x + 1\} > 0 \text{ for } 0 < x < 1 \tag{H4}$$

Equation (H4) means that for x, which is smaller than x^*, $\frac{\partial \alpha}{\partial x} < 0$; that is, α decreases. For x, which is larger than x^* such that $f_1(x^*) = 0$, $\frac{\partial \alpha}{\partial x} > 0$; that is, α increases. Thus, α^*, α associated with x^* that satisfies $f_1(x^*) = 0$, becomes the minimum value of α. Furthermore, $\frac{d\alpha}{d\gamma}$ is given by

$$\frac{d\alpha}{d\gamma} = \frac{\partial \alpha}{\partial x}\frac{dx}{d\gamma} = \frac{nf_1(x)}{\{1 + 2nx - (n+1)x^n\}^2}\frac{\overline{P}s - C(t^*)}{\Delta e}\left(\delta'_\gamma + g'_\gamma\right) \tag{H5}$$

From Equation (F20), $\delta'_\gamma + g'_\gamma > 0$ holds. Thus, $\frac{d\alpha}{d\gamma}$ associated with α^* becomes 0. Hence, α^* and its associated γ^* constitute the equilibrium solution. Furthermore, for γ, which is smaller than γ^*, $\frac{d\alpha}{d\gamma} < 0$; that is, α decreases. For γ, which is larger than γ^*, $\frac{d\alpha}{d\gamma} > 0$; that is, α increases. Items α^* and γ^* are obtained from Equations (44), (45), (42) and (43). The solution to satisfy the necessary conditions is summarised thus:

$$\frac{\partial C(t^*)}{\partial t} = 0 \tag{29''}$$

$$\beta^* = 0 \tag{36'}$$

$$\alpha^* = \left\{\frac{1 - x^{*n} + nx^*}{1 + 2nx^* - (n+1)x^{*n}}\right\}\{\overline{P}s - C(t^*)\} = x^*_\alpha\{\overline{P}s - C(t^*)\} \tag{H6), (44'}$$

$$\gamma^* = \left\{\frac{1 + 2nx^* - (n+1)x^{*n}}{(1 - x^{*n-1})n}\right\}\frac{\overline{P}(1-s)\Delta e}{\overline{P}s - C(t^*)} = x^*_\gamma \frac{\overline{P}(1-s)\Delta e}{\overline{P}s - C(t^*)} \tag{H7), (42'}$$

where

$$f_1(x^*) = (n-1)^2 x^{*n} + nx^{*n-1} - 1 = 0 \tag{H2'}$$

$$x^*_\alpha = \frac{1 - x^{*n} + nx^*}{1 + 2nx^* - (n+1)x^{*n}} \tag{H8), (45'}$$

$$x^*_\gamma = \frac{1 + 2nx^* - (n+1)x^{*n}}{(1 - x^{*n-1})n} \tag{H9), (43'}$$

Furthermore,

$$\alpha^*\gamma^* = \frac{1 - x^{*n} + nx^*}{(1 - x^{*n-1})n}\{\overline{P}(1-s)\Delta e\} = x^*_{\alpha\gamma}\{\overline{P}(1-s)\Delta e\} \tag{H10), (40'}$$

where

$$x^*_{\alpha\gamma} = \frac{1 - x^{*n} + nx^*}{(1 - x^{*n-1})n} \tag{H11), (41'}$$

The objective functions are given by:

$$O^*_1 = C(t^*) + \alpha^* = C(t^*) + x^*_\alpha\{\overline{P}s - C(t^*)\} \tag{H12}$$

$$O^*_2 = Prob{*}\alpha^* = Prob{*}x^*_\alpha\{\overline{P}s - C(t^*)\} \tag{H13}$$

where

$$Prob = p^* = \frac{1}{n}\left(1 - x^{*^n}\right) + x^* \tag{H14}$$

The solution to satisfy the necessary conditions in Scenario without LB, denoting $E_C(t^*, \alpha^*, \beta^*, \gamma^*)$, is summarised in (46).

The result shows the solutions to Equations $(H2')$and (47), x^*, and x^*_α, x^*_γ, and $x^*_{\alpha\gamma}$ between $n = 2$ and $n = 20$, considered the realistic range of bidders. These values only depend on n, the number of bidders. (See detailed data in Table H1 in the Supplemental online material).

Q.E.D.

I. Proof of Theorem 6

We put superscript, *, on each variable at equilibrium.

Inequality (38) can be transformed as:

$$u_1 = \frac{1}{\gamma^*}\left(\frac{J^*_1}{J^*_2}\right) - 2\left(\frac{J^*_1}{J^*_2}\right)'_\gamma = \frac{1}{\gamma^* J^*_2 2}\left(J^*_1 J^*_2 - 2\gamma J^*_{3\gamma}\right) \le 0 \tag{I1}$$

Thus, this inequality is equivalent to

$$J^*_1 J^*_2 - 2\gamma^* J^*_{3\gamma} \le 0 \tag{I1'}$$

First, we demonstrate that the J^*_1, J^*_2 and $J^*_{3\gamma}$ values only depend on the value of x^* and n, the number of bidders.

Transforming Equations (H10) and (40′) yields

$$x^*_{\alpha\gamma} = \frac{\alpha^*\gamma^*}{\overline{P}(1-s)\Delta e} = \frac{-h\alpha^*\gamma^*}{\Delta e} = \frac{-g^*}{\Delta e} \tag{I2}$$

Thus, we obtain

$$J^*_1 = 1 - x^{*^{n-1}} - \frac{(n-1)}{\Delta e}g^* x^{*^{n-2}} = 1 - x^{*^{n-1}} + (n-1)x^*_{\alpha\gamma}x^{*^{n-2}} \tag{I3}$$

$$J^*_2 = 1 - x^{*^{n-1}} - \frac{(n-1)}{2\Delta e}g^* x^{*^{n-2}} = 1 - x^{*^{n-1}} + \frac{(n-1)}{2}x^*_{\alpha\gamma}x^{*^{n-2}} \tag{I4}$$

From Equations (D18), (D17) and (37), at equilibrium,

$$\delta^{*\prime}_\gamma = h\{C(t^*) - \overline{P}s\} \tag{I5}$$

$$g^{*\prime}_\gamma = h\alpha^* \tag{I6}$$

$$\delta^{*\prime}_\gamma + g^{*\prime}_\gamma = h\{C(t^*) - \overline{P}s\} + h\alpha^* \tag{I7}$$

For $n = 2$, from Equations (F14), (I5), (I6) and (I7),

$$2\gamma^* J^*_{3\gamma} = -\frac{\gamma^*}{\Delta e}\left\{\frac{1}{\Delta e}g^*\left(\delta^{*'}_\gamma + g^{*'}_\gamma\right) + (1 - x^*)g^{*'}_\gamma\right\}$$

Substituting Equations (37) yields

$$2\gamma^* J^*_{3\gamma} = -\left[\left\{-2\left(\frac{J^*_2}{J^*_1}\right)\frac{h\alpha^*\gamma^* g^*}{\Delta e^2} + \frac{g^{*^2}}{\Delta e^2}\right\} + (1 - x^*)\frac{g^*}{\Delta e}\right]$$
$$= -\left[\left\{-2\left(\frac{J^*_2}{J^*_1}\right) + 1\right\}\frac{g^{*^2}}{\Delta e^2} + (1 - x^*)\frac{g^*}{\Delta e}\right]$$
$$= -\left[\left\{-2\left(\frac{J^*_2}{J^*_1}\right) + 1\right\}x^*_{\alpha\gamma}2 - (1 - x^*)x^*_{\alpha\gamma}\right] \tag{I8'}$$

For $n \ge 3$, from Equations (F16), (I5), (I6) and (I7),

$$2\gamma^* J^*_{3\gamma} = \frac{-(n-1)x^{*^{n-3}}\gamma^*}{\Delta e}\left[\left\{(n-2) + x^{*^{n-1}}\right\}\frac{g^*}{\Delta e}\left(\delta^{*'}_\gamma + g^{*'}_\gamma\right) + x^*\left(1 - x^{*^{n-1}}\right)g^{*'}_\gamma\right]$$

$$= -(n-1)x^{*^{n-3}}\left[\{(n-2)+x^{*^{n-1}}\}\left\{\frac{h\gamma^* g^*}{\Delta e^2}(C(t^*)-\bar{P}s)+\frac{g^{*^2}}{\Delta e^2}\right\}+x^*(1-x^{*^{n-1}})\frac{g^*}{\Delta e}\right] \tag{I9}$$

Substituting Equations (37) yields

$$2\gamma^* J^*_{3\gamma} = -(n-1)x^{*^{n-3}}\left[\{(n-2)+x^{*^{n-1}}\}\left\{-2\left(\frac{J_2^*}{J_1^*}\right)\frac{h\alpha^*\gamma^* g^*}{\Delta e^2}+\frac{g^{*^2}}{\Delta e^2}\right\} + x^*(1-x^{*^{n-1}})\frac{g^*}{\Delta e}\right]$$

$$= -(n-1)x^{*^{n-3}}\left[\{(n-2)+x^{*^{n-1}}\}\right.$$

$$\left.\left\{-2\left(\frac{J_2^*}{J_1^*}\right)+1\right\}x^*_{\alpha\gamma}2 - x^*(1-x^{*^{n-1}})x^*_{\alpha\gamma}\right] \tag{I9'}$$

Hence, the values of u_1 and u_2 described in Equations (38) and (39), which consist of J_1^*, J_2^* and $2\gamma^* J^*_{3\gamma}$, only depend on n.

Next, we numerically demonstrate that sufficient conditions are satisfied. The result shows the values of u_1 and u_2 and the satisfaction of sufficiency conditions between $n=2$ and $n=20$. For $n \geq 4$, $u_1 \leq 0$ is satisfied. Thus, the sufficient condition is satisfied. (See detailed data in Table I1 in the Supplemental online material).

Q.E.D.

J. Proof of Lemma 4

We examine the characteristics of the direct first and the second derivatives of $\alpha\gamma$ with respect to γ. Equation (F6″) is again given by

$$\frac{d(\alpha\gamma)}{d\gamma} = \frac{1}{2}\frac{J_1}{J_2}(\bar{P}s - c(t)) > 0 (F6'')$$

From Equations (F10), the relationship below holds.

$$\frac{d^2(\alpha\gamma)}{d\gamma^2} = \frac{1}{2}\left\{\left(\frac{J_1}{J_2}\right)'_\gamma(\bar{P}s-c(t)) - \left(\frac{J_1}{J_2}\right)\frac{\partial C(t)}{\partial t}\frac{dt}{d\gamma}\right\}$$

$$= \frac{1}{2}\left(\frac{J_1}{J_2}\right)'_\gamma(\bar{P}s-c(t)) > 0 \tag{J1}$$

First, $q_1(\gamma) = \alpha\gamma$ represents the relationship between the products of $\alpha\gamma$ and γ that satisfy xxx Equations (29′), (22″), (36) and (11″). Equation (37) holds in equilibrium:

$$\frac{d\alpha}{d\gamma} = 0 \tag{37}$$

This implies

$$\frac{d(\alpha\gamma)}{d\gamma} = \alpha + \gamma\frac{d\alpha}{d\gamma} = \alpha \tag{J2)}$$

where $q_2(\gamma)$ represents the tangent of $q_1(\gamma)$. Equation (J2) means that, in the equilibrium solution, the tangent of $q_2(\gamma)$ becomes a straight line with a slope α^* that passes through the origin and $(\gamma^*, \alpha^*\gamma^*)$. This represents the physical interpretations of γ and α. Q.E.D.

K. Proof of Theorem 7

From Equation F(20), $\delta'_\gamma + g'_\gamma > 0$ holds; when γ takes the minimum value, $\delta + g$ becomes the minimum, and vice versa. Thus, when $x = \frac{\delta+g}{\Delta e} = 0$, $\gamma = \gamma_{\min}$.

Setting $\gamma_{\min} = \gamma_o$ and denoting x_{γ_o}, x_{α_o} and α_o, which are x_γ, x_α and α associated with γ_o. From Equations (42), (43), (44) and (45), we obtain the following equations.

$$x_{\gamma_o} = \frac{1}{n} \tag{K1)}$$

$$\gamma_o = x_{\gamma_o}\frac{\bar{P}(1-s)\Delta e}{\{\bar{P}s - C(t^*)\}} = \frac{\bar{P}(1-s)\Delta e}{\{\bar{P}s - C(t^*)\}n} \tag{K2), (48}$$

$$x_{\alpha_o} = 1 \tag{K3}$$

$$\alpha_0 = \bar{P}s - C(t^*) \tag{K4), (50}$$

Next, we inquire into the maximum value of x_γ and γ.

$$\lim_{x\to 1} x_\gamma = \lim_{x\to 1}\left\{\frac{1+2nx-(n+1)x^n}{(1-x^{n-1})n}\right\} = \lim_{x\to 1}\left\{\frac{1}{1-x^{n-1}}\right\} = \infty$$

$$= \text{Sup}x_\gamma \tag{K5}$$

$$\lim_{x\to 1}\gamma = \lim_{x\to 1} x_\gamma\left[\frac{\bar{P}(1-s)\Delta e}{\{\bar{P}s - C(t^*)\}}\right] = \infty = \text{Sup}\gamma \tag{K6), (49}$$

At this point, x_α and α are given by

$$\lim_{x\to 1} x_\alpha = \lim_{x\to 1}\left\{\frac{1-x^n+nx}{1+2nx-(n+1)x^n}\right\} = 1 \tag{K7}$$

$$\lim_{x\to 1}\alpha = x_\alpha\{\bar{P}s - C(t^*)\} = \{\bar{P}s - C(t^*)\} = \alpha_0 \tag{K8}$$

Regarding the behaviour of α, that is, decreasing for $\gamma < \gamma^*$, taking the minimum value, α^*, when $\gamma = \gamma^*$, and increasing for $\gamma > \gamma^*$, was demonstrated in Appendix H. From $(K7)$ and $(K8)$, α approaches α_o as γ increases.

Q.E.D.

L. Proof of Theorem 8

We assume that β^* and t^* satisfy the first-order conditions.

$$\frac{\beta^* t^*}{\Delta e} = 1 \tag{L1), (55'}$$

$$\frac{\partial C(t^*)}{\partial t} = 0 \tag{L2), (56'}$$

We obtain the Hessian matrix by taking the second partial derivatives of O_2 with respect to β and t. Those values associated with β^* and t^* are given as follows:

$$\frac{\partial^2 O_2}{\partial\beta^2} = -(n-1)\left(\frac{t^*}{\Delta e}\right)^2\{\bar{P}s - C(t^*)\} < 0 \tag{L3}$$

$$\frac{\partial^2 O_2}{\partial t^2} = -(n-1)\left(\frac{\beta}{\Delta e}\right)^2\left(\frac{\beta t}{\Delta e}\right)^{n-2}(\bar{P}s - C(t))$$

$$-\frac{2\beta}{\Delta e}\left\{1-\left(\frac{\beta t}{\Delta e}\right)^{n-1}\right\}\frac{\partial C(t)}{\partial t}$$

$$-\left[\frac{1}{n}\left\{1-\left(\frac{\beta t}{\Delta e}\right)^n\right\}+\frac{\beta t}{\Delta e}\right]\frac{\partial^2 C(t)}{\partial t^2}$$

$$= -(n-1)\left(\frac{\beta^*}{\Delta e}\right)^2(\bar{P}s - C(t^*)) - \frac{\partial^2 C(t^*)}{\partial t^2} < 0 \tag{L4}$$

$$\frac{\partial^2 O_2}{\partial\beta\partial t} = \frac{1}{\Delta e}\left[\left\{1-\left(\frac{\beta t}{\Delta e}\right)^{n-1}\right\} - (n-1)\left(\frac{\beta t}{\Delta e}\right)^{n-1}\right](\bar{P}s - C(t))$$

$$-\frac{t}{\Delta e}\left\{1-\left(\frac{\beta t}{\Delta e}\right)^{n-1}\right\}\frac{\partial C(t)}{\partial t}$$

$$= -\frac{1}{\Delta e}(n-1)\left(\overline{P}s - C(t^*)\right) \quad \text{(L5)}$$

The Hessian matrix of O_2 is given by:

$$H_{O_2} = \begin{pmatrix} \frac{\partial^2 O_2}{\partial \beta^2} & \frac{\partial^2 O_2}{\partial \beta \partial t} \\ \frac{\partial^2 O_2}{\partial \beta \partial t} & \frac{\partial^2 O_2}{\partial t^2} \end{pmatrix} \quad \text{(L6)}$$

Its determinant is given by:

$$det = \frac{\partial^2 O_2}{\partial \beta^2}\frac{\partial^2 O_2}{\partial t^2} - \left(\frac{\partial^2 O_2}{\partial \beta \partial t}\right)^2$$

$$= \frac{1}{\Delta e^2}(n-1)^2\left(\frac{\beta^* t^*}{\Delta e}\right)^2\left(\overline{P}s - C(t^*)\right)^2$$
$$+ (n-1)\frac{\partial^2 C(t^*)}{\partial t^2}\left(\frac{t^*}{\Delta e}\right)^2\left(\overline{P}s - C(t^*)\right)$$
$$-\left\{\frac{1}{\Delta e}(n-1)\left(\overline{P}s - C(t^*)\right)\right\}^2$$
$$= (n-1)\frac{\partial^2 C(t^*)}{\partial t^2}\left(\frac{t^*}{\Delta e}\right)^2\left(\overline{P}s - C(t^*)\right) > 0 \quad \text{(L7)}$$

Hence, t^* satisfies sufficient conditions for Equations (L3), (L4) and (L7). Thus, O_2 is locally maximised at $\beta = \beta^*$ and $t = t^*$.

Q.E.D.